Nonconvex Generalized Benders Decomposition for Natural Gas Production Network Design and Operation Under Uncertainty

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Natural gas production networks involve gas wells in different reservoirs, transmission pipelines, gas production platforms, riser platforms, simple mixing and splitting units and gas terminals (such as liquefied natural gas plants or dry gas processing and distribution terminals) that process gas to supply markets. The integrated design and operation of such networks determines simultaneously the network development decisions and the operating conditions that can achieve the best expected profitability while ensuring the minimum supply of gas to markets and the satisfaction of product specific requirements. Due to the need to track gas qualities (i.e., compositions) and address the large uncertainties affecting the decisions, this problem is formulated as a two-stage stochastic nonconvex mixed-integer nonlinear program (MINLP) with recourse.

Due to their special structures, stochastic programs with recourse have long been solved with duality-based decomposition strategies, typically the Benders decomposition/*L*-shaped method [1] [2] and generalized Benders decomposition [3]. However, these methods cannot solve nonconvex stochastic programs rigorously because of duality gap. This paper presents a rigorous, duality-based decomposition method, called nonconvex generalized Benders decomposition (NGBD), for stochastic nonconvex MINLP, which guarantees finding an ε-optimal solution within a finite number of iterations. The method follows the framework of concepts presented by Geoffrion [4] for the design of large-scale mathematical programming techniques and the notion of generalized Benders decomposition. By convexification of the nonconvex functions, the stochastic MINLP is relaxed into a lower bounding problem that is a potentially large-scale convex MINLP. Solution of this lower bounding problem is then decomposed into a sequence of relaxed master problems, which are mixed-integer linear programs (MILP) with much smaller sizes, and primal bounding problems, which are convex programs. The solutions of the relaxed master problems yield a sequence of nondecreasing lower bounds on the optimal objective value, and they also generate a sequence of integer realizations defining the primal problems which yield a sequence of nonincreasing upper bounds on the optimal objective value. It can be proved that NGBD terminates finitely when the lower and upper bounds coincide (or are close enough), or infeasibility of the problem is indicated.

The dramatic computational advantages of NGBD over the state-of-the-art branch-and-reduce global optimization method (by BARON [5]) will be demonstrated via case studies of a real industrial system, the Sarawak Gas Production System [6]. Two types of design problems are considered. The first one addresses the gas qualities but ignores the pressure-flow relationship in the production network, so the nonconvex constraints involved in the problem are all bilinear. A problem of this type with almost 100,000 variables can be solved by NGBD within 1000 seconds. The second one includes a more sophisticated model that also addresses the pressure-flow relationship and the compressor performance model, so the problem also includes equations with quadratic and power functions. A problem of this type with almost 150,000 variables can be solved within 80 minutes.

References

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