An Integer Linear Programming apporach to AC power grid design*

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Abstract

A power grid is a transmission network transporting electrical energy from power plants to some substations near urban or industrial centers. To reduce the loss of energy, different high voltages (at least 110kV) and alternating currents (AC) are used to transport the power. The AC power flow consists of two single power flows called active and reactive flow.

The network consists of different subnetworks each with its own level of voltage, whereby a single network contains nodes with a specific demand of active and reactive power which are connected through lines. The nodes may represent substations which lead to different networks of a lower voltage level or groups of customers. The lines may represent underground power cables or overhead power lines.

In this work, we consider the design of AC power grids, including the placement of supply equipment (called generators). The potential topology is modelled as an undirected graph $N = (V, E)$, where the set V denotes the demand nodes and E the set of all possible lines between the nodes. The design problem is to find the minimum cost network which fulfills all demands. Given a selection of the lines, we need to calculate the power flow in the network. In an AC network, we have an active and a reactive power flow which periodically reverse their direction. The computation of these bidirectional power flows involves complex numbers and nonlinear functions.

The most common way to handle the nonlinearities is to use the *DC model* which provides linear approximations. Although the DC modelling of the power flow has proven to be very fast, its major drawback is that information about the reactive flows is lost. Additionally, we loose all information about accuring power losses, as the forward and backward power flows are equal.

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In view of recent developments in integer linear programming, we revisit a less known linearization which approximates both the active and the reactive power flow. In our linearization the power flow will be symmetric (like in the *DC model*), guaranteeing that the problem can be easier solved.

Finally, we will compare both power flow models in the context of power grid design with an MILP model which aims to construct the optimal power grid with respect to installation costs. The alternative linearization yields a more accurate approximation of the non-linear power flow, but is signficantly slower than the respective DC formulation.

References

[1] Andersson, G.: Modelling and Analysis of Electric Power Systems (2008). URL http://www.eeh.ee.ethz.ch/uploads/tx_ethstudies/modelling_hs08_script_02.pdf. ETH Zürich

[2] Braun, A.: Anlagen- und Strukturoptimierung von 110-kV-Netzen. Ph.D. thesis, RWTH Aachen University (2001)

[3] Maurer, H.C.G.: Integrierte Grundsatz- und Ausbauplanung für Hochspannungsnetze. Ph.D. thesis, RWTH Aachen University (2004)

[4] Moser, A.: Langfristig optimale Struktur und Betriebsmittelwahl für 110-kV-Überlandnetze. Ph.D. thesis, RWTH Aachen University (1995)