The Deployment of Fast A* Search
CPAIOR Master Class

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Outline

- Fundamentals
  - Best-first search & heuristics
- Abstraction
  - Deployment in commercial software
- Heuristics
  - Heuristics and Maximum Variance Unfolding

Best-First Search

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Problem Size Comment

- Generally referring to interesting class of problems:
  - The faster search the better
  - Small enough to fit in memory
  - Not enough memory for all-pairs shortest paths
  - *Solving different instances on same graph each time*
- Examples:
  - Maps / navigation
  - Video games

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State Space Search

- We assume:
  - A start state
  - A successor function
  - A goal state or a goal test function

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Algorithm Measures

- Complete
  - Is it guaranteed to find a solution if one exists
- Optimal
  - Is it guaranteed the find the optimal solution

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Best-First Algorithms

- Choose a metric of best
  - Expand states in order from best to worst

- Requires:
  - Sorted OPEN list/priority queue
  - CLOSED list

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Definitions

- Node is *expanded* when taken off queue
- Node is *generated* when put on queue

- *g*-cost is the cost from the start to the current node
- *c*(*a*, *b*) is the edge cost between *a* and *b*
  - Sometimes also designates optimal path cost

Best-First Algorithms

- Best-First Pseudo-Code
  1. Put start on OPEN
  2. While(OPEN is not empty)
     1. Pop best node *n* from OPEN
     2. if (*n* == *goal*) return path(*n*, *goal*)
     3. for each child of *n* // generate children
        1.put/update value on OPEN/CLOSED
  3. return NO PATH

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Best-First Algorithms: Updating children

- Best-First child update
  1. If child on OPEN, and new cost is less
     1. Update cost and parent pointer
  2. If child on CLOSED
     1. Ignore*
  3. Otherwise
     1. Add to OPEN list

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Uniform-Cost Search

- Dijkstra's algorithm
  - Best-first is the *g*-cost

Dijkstra - Detail

Dijkstra

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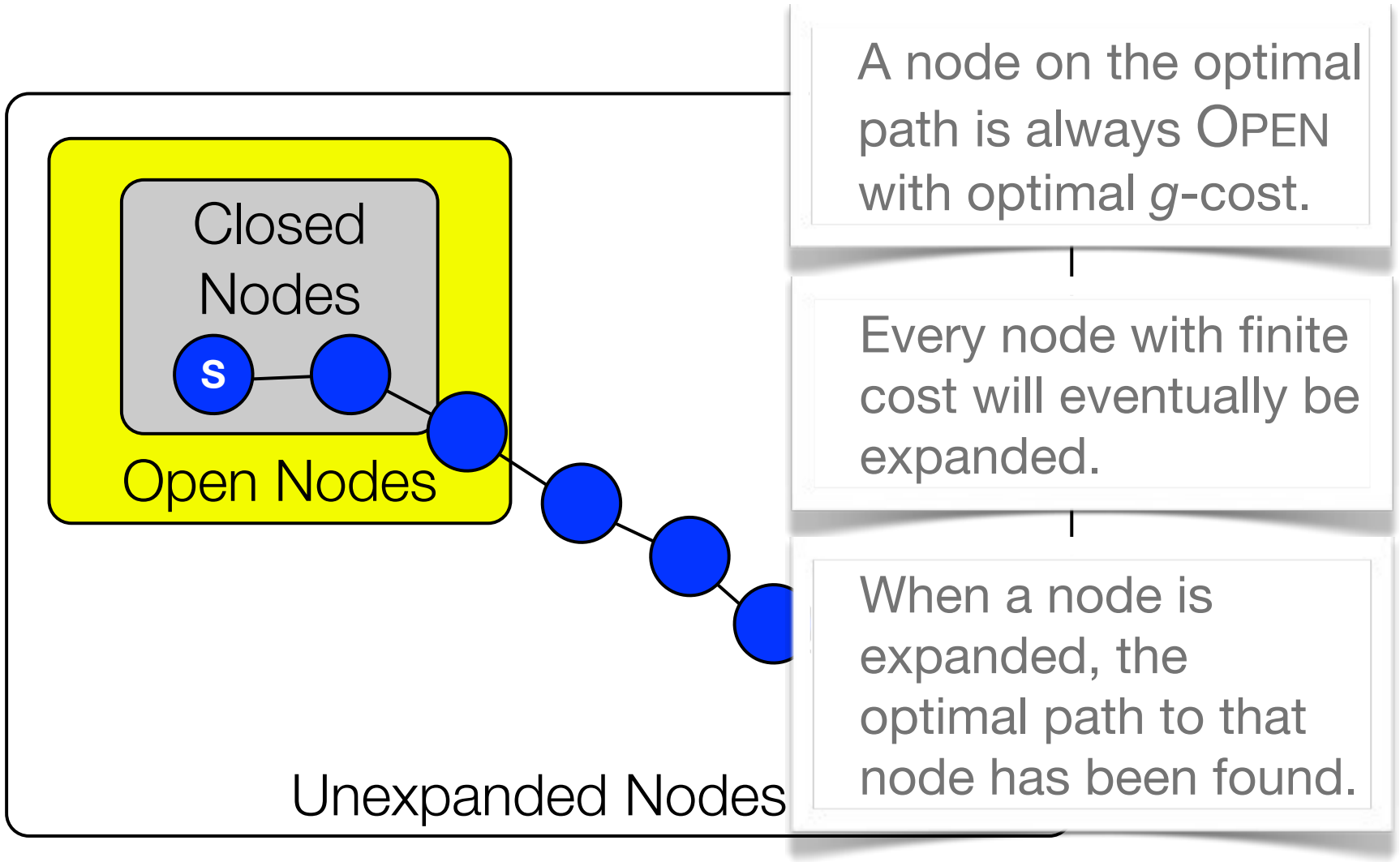
Dijkstra's Completeness

- Complete? / Will it find a solution?
  - Finite Graph: yes
  - Infinite Graph
    - Must have finite cost path to goal
    - Edge costs at least epsilon
    - Cannot have negative cost loops

Dijkstra's Optimality: High Level View



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Dijkstra's Time & Space Complexity

- In exponential domains assumed:
  - Branching factor *b*
  - Minimum edge cost *e*
  - $O(b^{c/e})$ -- *c* - solution cost
- In AI we ignore data structure overheads
  - Assume that costs can be bucketed
- Harder to analyze domains that fit in memory

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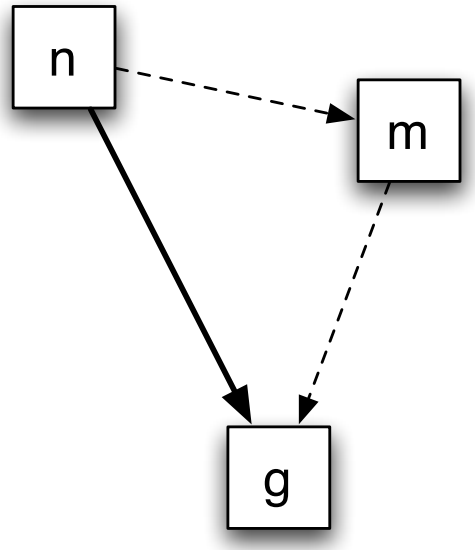
Heuristic

- What is a heuristic?
  - An estimate of the cost from a given state to the goal
- Where do they [traditionally] come from?
  - Solutions to abstracted problem

Properties of Heuristics

- Perfect heuristic: h*(n)
- Admissible: h(n) ≤ h*(n) for all n
- Consistent:
  - h(n) ≤ c(n, m) + h(m) (directed)
  - c(n, m) ≥ |h(n) - h(m)| (undirected)



- Local consistency implies global consistency
- Consistency implies admissibility

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f-cost Monotonically Non-decreasing

- *f*-cost: *g-cost* + *h*-cost
- $h(n) \leq h(m) + c(n, m)$ [*consistency*]
- $g(n) + h(n) \leq g(n) + h(m) + c(n, m)$
- $f(n) \leq g(m) + h(m) = f(m)$

- *f*-cost is monotonic non-decreasing

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Pure Heuristic Search / Greedy Best-first Search

- Best-first is the *h*-cost
- Complete?
  - Only on finite graph
- Optimal?
  - No

Pure Heuristic Search

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A*

- Best-first is the *f*-cost
  - $f = g + h$
  - *f* is an estimate of the complete path length
- Optimality?
  - Depends on the heuristic

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A* Completeness

- Complete? / Will it find a solution?
  - Finite Graph: yes
  - Infinite Graph
    - Must have finite cost path to goal
    - Edge costs at least epsilon
    - Cannot have negative cost loops
  - Finite heuristic cost

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A* Optimality

- Same conditions as Dijkstra
- Plus one of:
  - Consistent Heuristic
  - Admissible heuristic
    - Need to change child update rule

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A* - Analysis

- Worst case: h(n) = 0 for all *n*
  - Same as Dijkstra $O(b^{c/e})$
- Best case: h(n) = h*(n) for all *n*
  - Will go straight to the goal(?)

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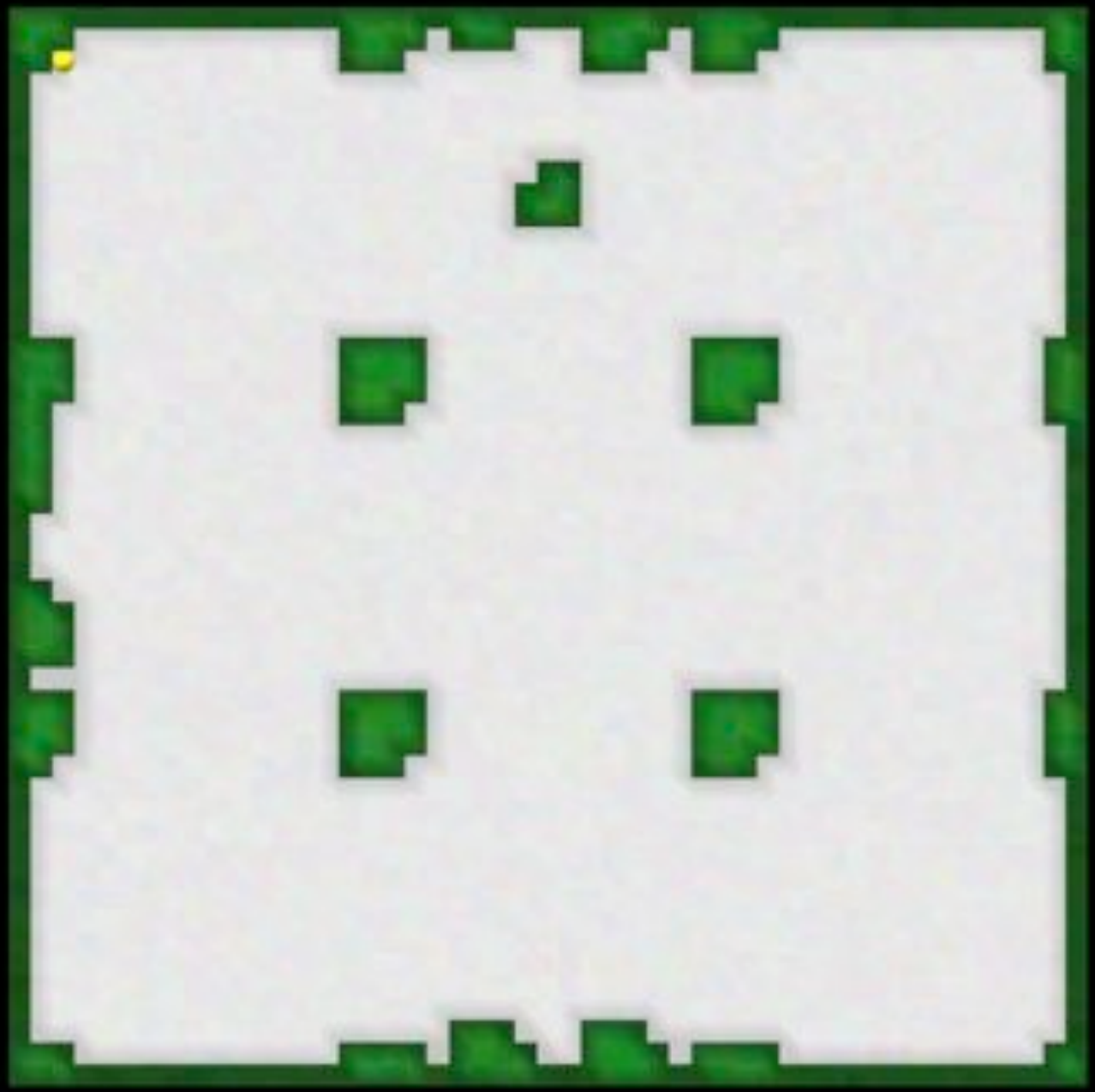
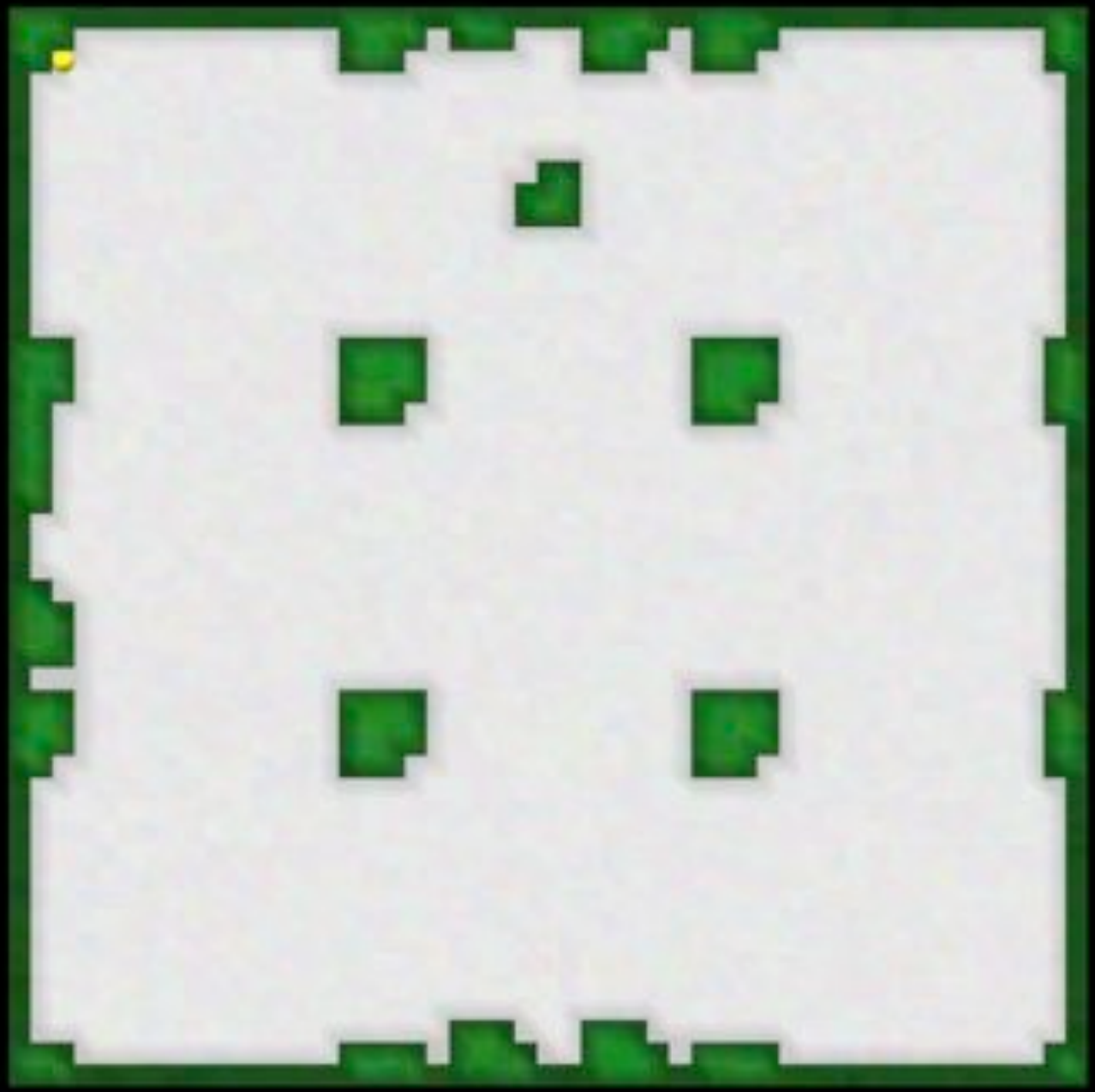
A* implementation details

- Break ties towards states with higher g-cost
- If a successor has *f*-cost as good as the front of OPEN
  - Avoid the sorting operations
- Make sure heuristic matches problem representation
  - With 8-connected grids don't use straight-line heuristic

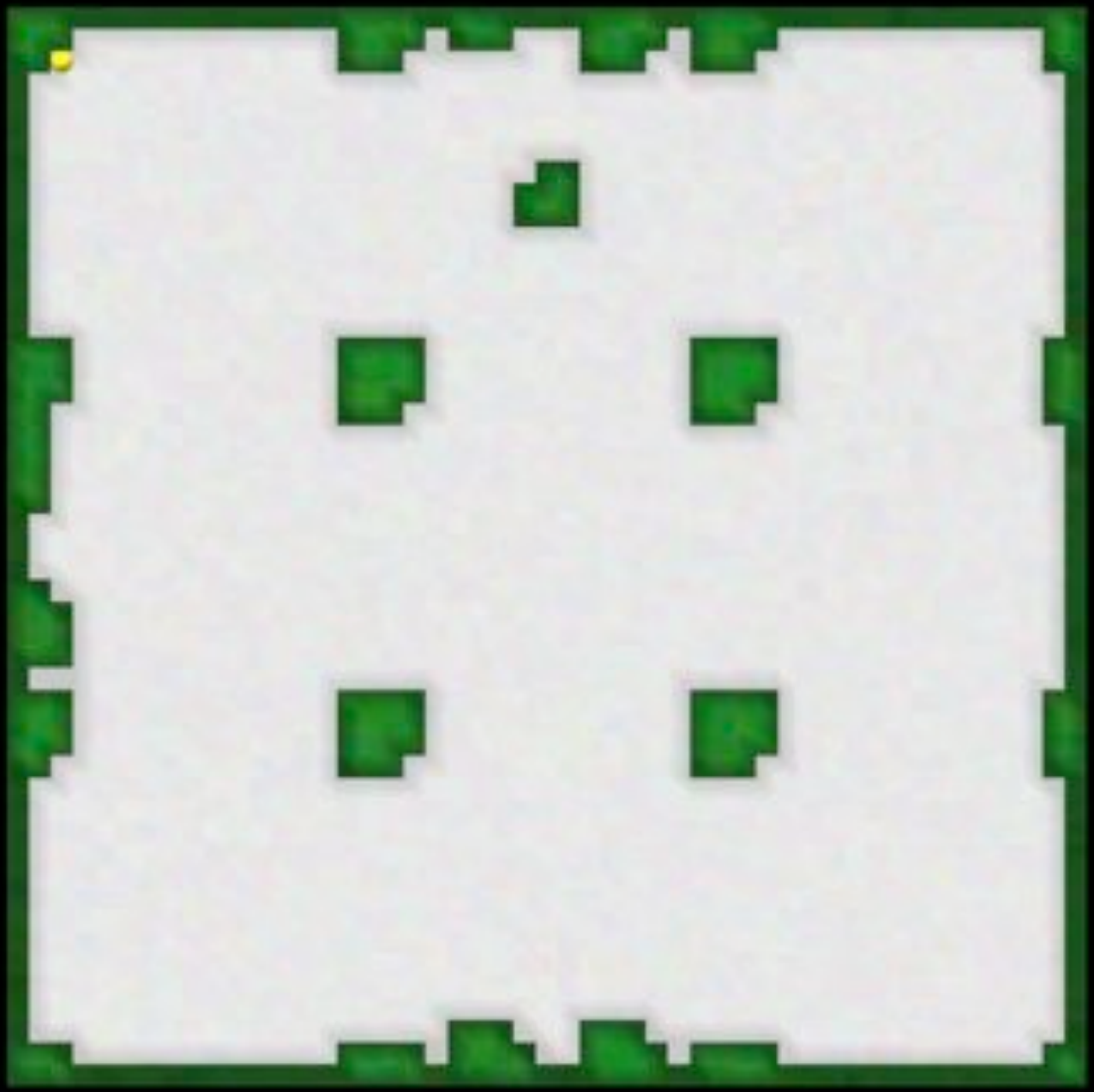
A* - Ties towards high g-costs



A* - Ties towards low g-costs



Heuristic: Euclidean Distance



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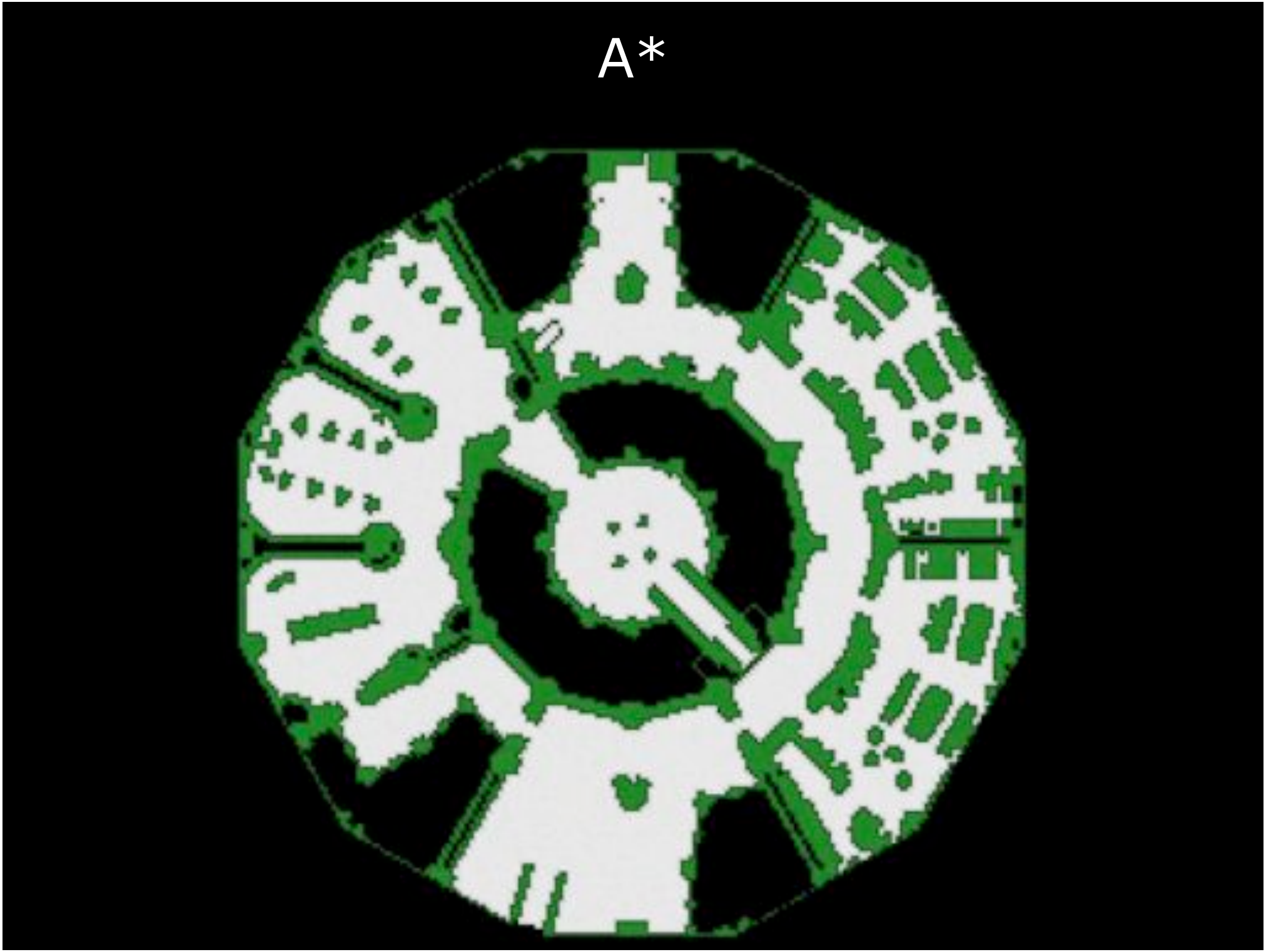
A* is Optimal in Unique Node Expansions

- Suppose there was a better algorithm, E
  - There must be some node *n* which A* expands, but E doesn't
  - Re-arrange the problem and put the goal after *n*
  - E cannot be optimal/complete

A*



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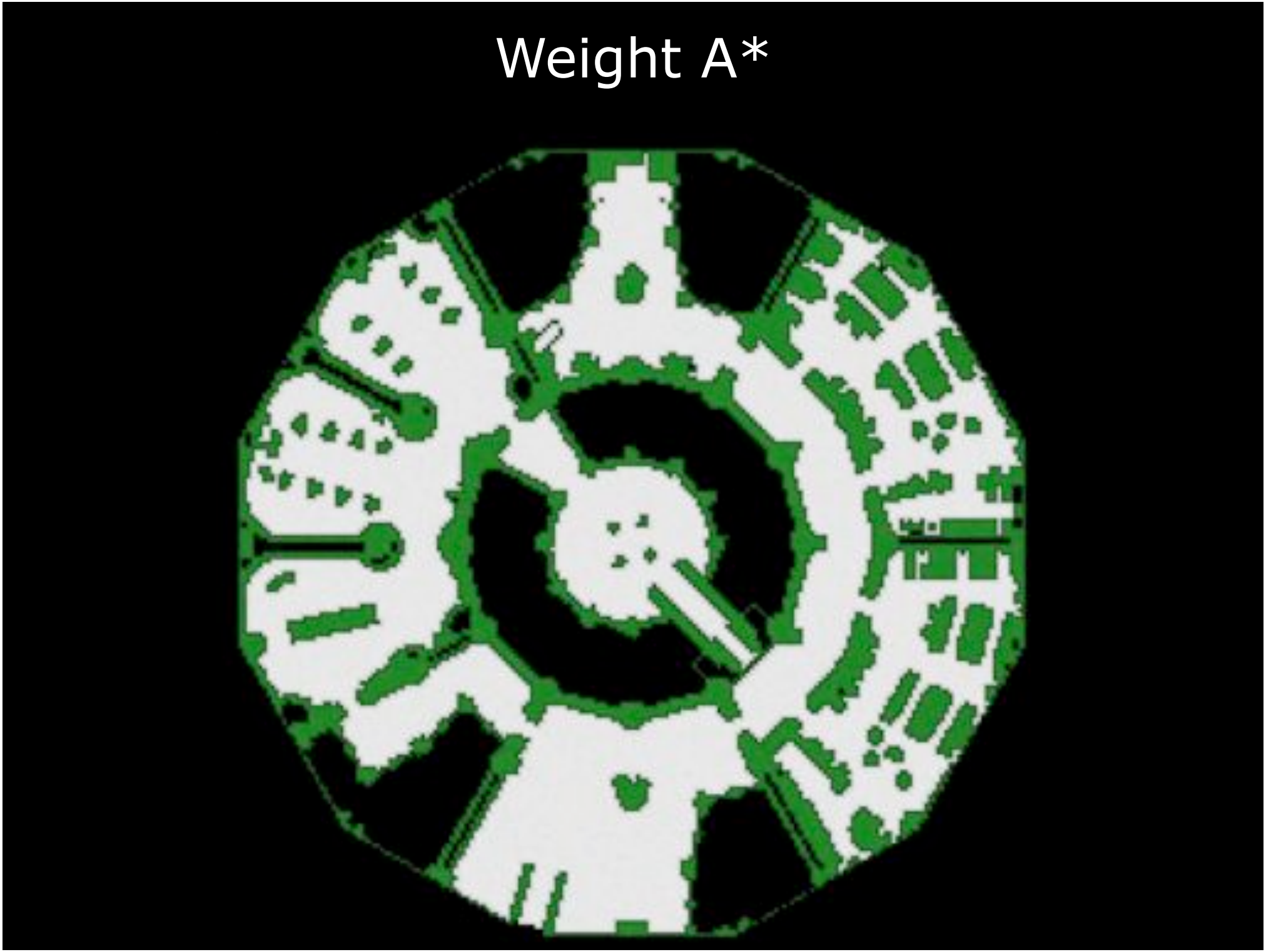
Can we make A* go faster?

- A*: f(n) = g(n) + h(n)
- Weighted A*: f(n) = (1-*w*)·g(*n*) + *w*·h(*n*)
- If *w* = 1?
  - Pure Heuristic Search
- If *w* = 0?
  - Dijkstra's
- Similar to depth-bounded discrepancy search

Weight A*



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IDA*

- IDA* is an iterative deepening version of A*
  - Very useful in exponential domains
  - Cost of iterations is completely amortized in search
- Not used on problems with many cycles
- Similar to Limited Discrepancy Search

Inconsistent Heuristics

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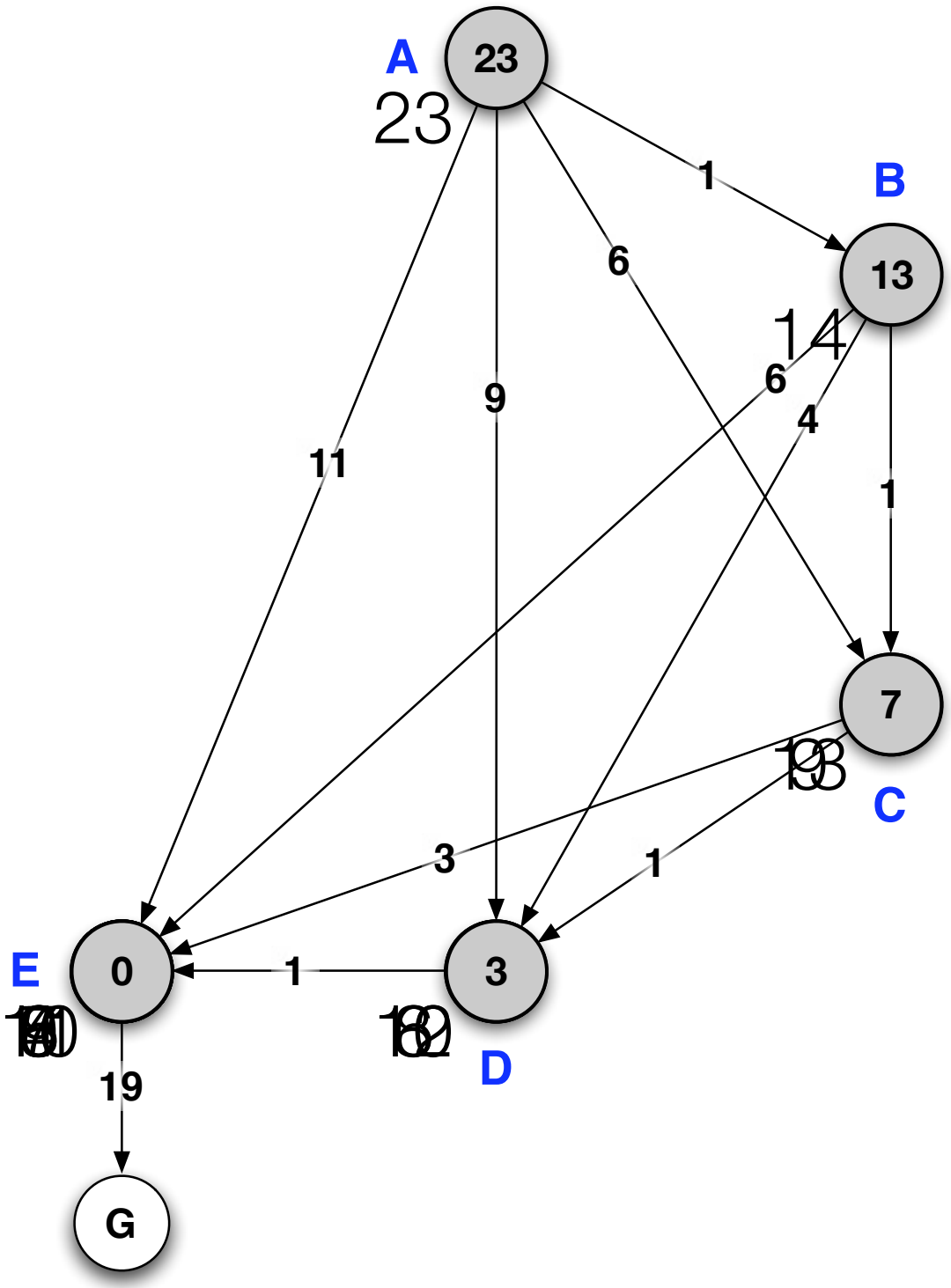
When A* doesn't work well

- Inconsistent heuristics ignored for ~20 years
  - Not thought to occur in practice

- What happens when we have inconsistent heuristics?
  - Optimal path to goal will still be found
  - First path to a node is not necessarily optimal
  - Can re-open closed nodes



node	f	g	h
A	23	0	23
E	11	11	0
D	12	9	3
E	10	10	0
C	13	6	7
E	9	9	0
D	10	7	3
E	8	8	0
B	14	1	13
E	7	7	0
D	8	5	3
E	6	6	0
C	9	2	7
E	5	5	0
D	6	3	3
E	4	4	0
G	23	23	0

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Definition

- Let N be the number of nodes expanded by A*
  - N is the number of *nodes* not the number of *expansions*
- A* can re-expand nodes
  - Express in terms of N

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Analysis

- With N nodes, $(1 + 2^{N-2})$ nodes expanded!
  - 6 nodes
  - “E” expanded 8 times
  - “D” expanded 4 times
  - “C” expanded 2 times
  - “A”, “B”, “G” expanded once

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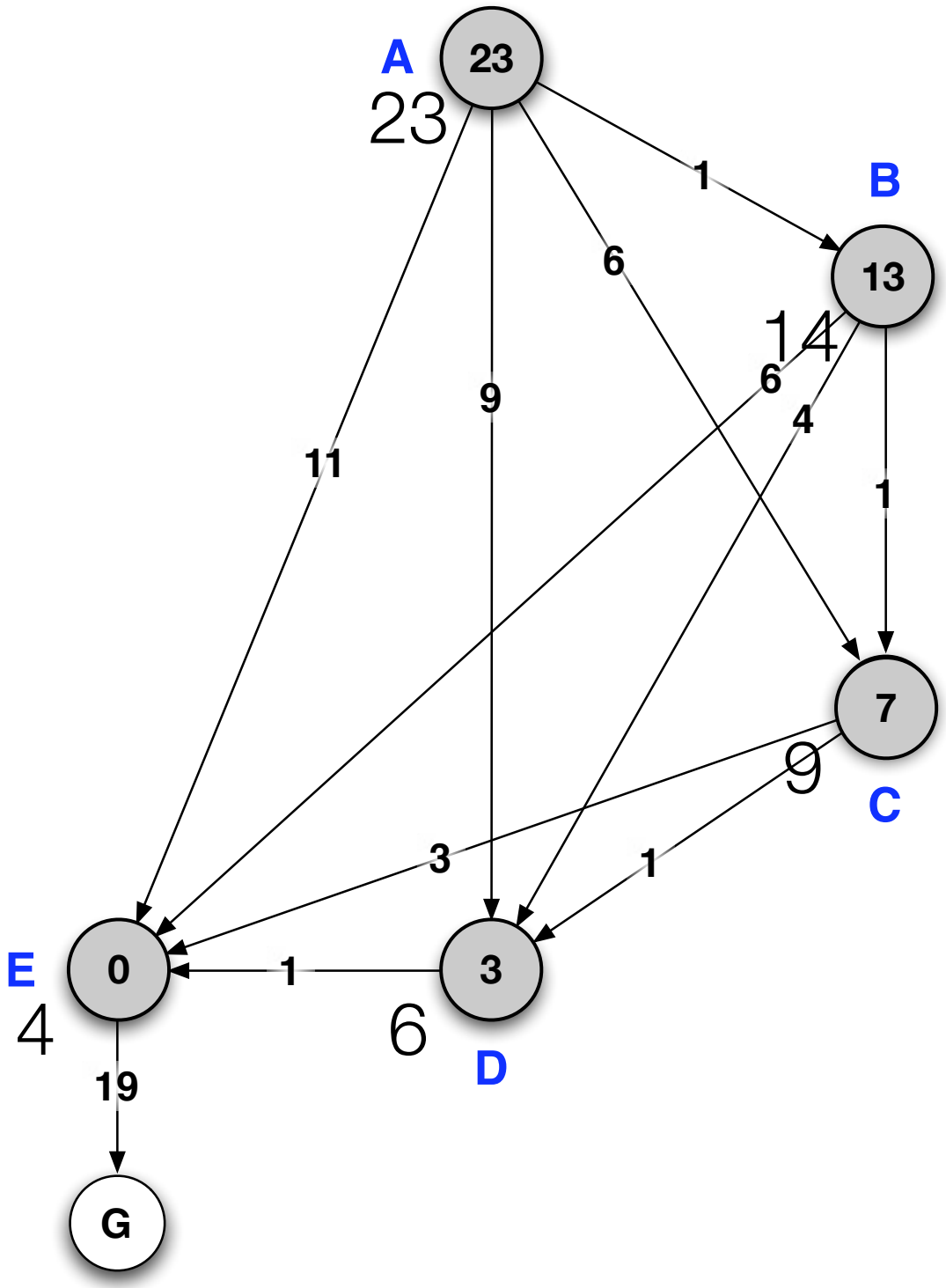
Martelli, 1979

- Suggested Algorithm “B”
  - Maintain global “F” value
  - Maximum f-value opened so far
  - If there are nodes on OPEN with f < F
    - Open in order of increasing g-cost
    - Dijkstra’s algorithm



node	f	g	h
A	23	0	23
B	14	1	13
C	9	2	7
D	6	3	3
E	4	4	0
G	23	23	0

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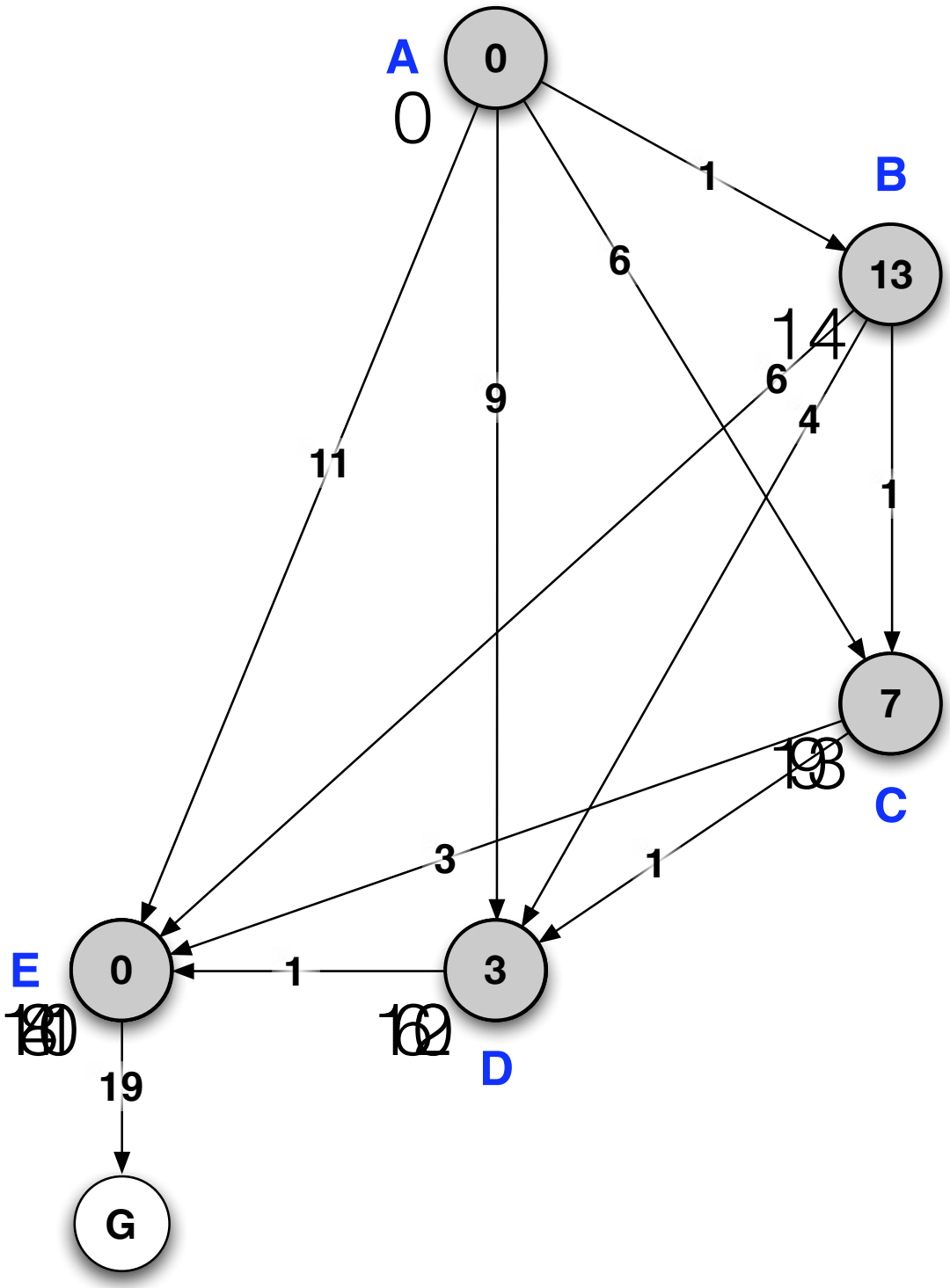
Martelli

- Algorithm works well, does it fix everything?
  - No -- worst case still $O(N^2)$
  - Just lower cost of start heuristic to 0



node	f	g	h	F
A	0	0	0	0
E	11	11	0	11
D	12	9	3	12
E	10	10	0	12
C	13	6	7	13
D	10	7	3	13
E	8	8	0	13
B	14	1	13	14
C	9	2	7	14
D	6	3	3	14
E	4	4	0	14
G	23	23	0	23

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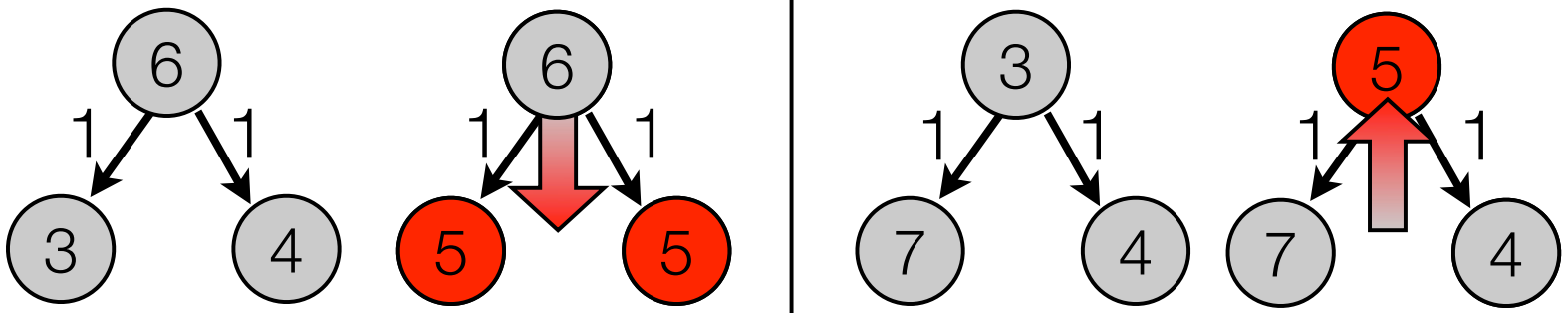
Solution - Mero 84 - B’

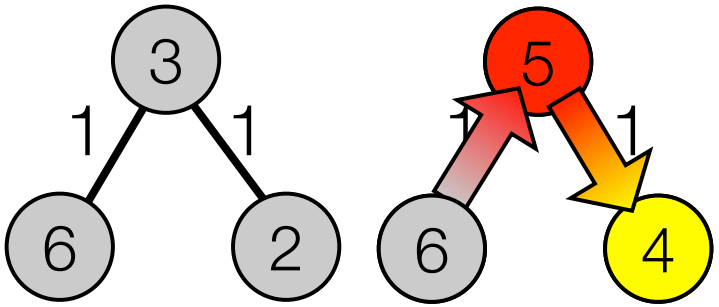
- Pathmax
  - When generating a node:
  - h’(n) = max(h(p) - c(n, p), h(n))
  - h’(p) = max(h(p), min(h(c) + c(c, p))) over all children *c*

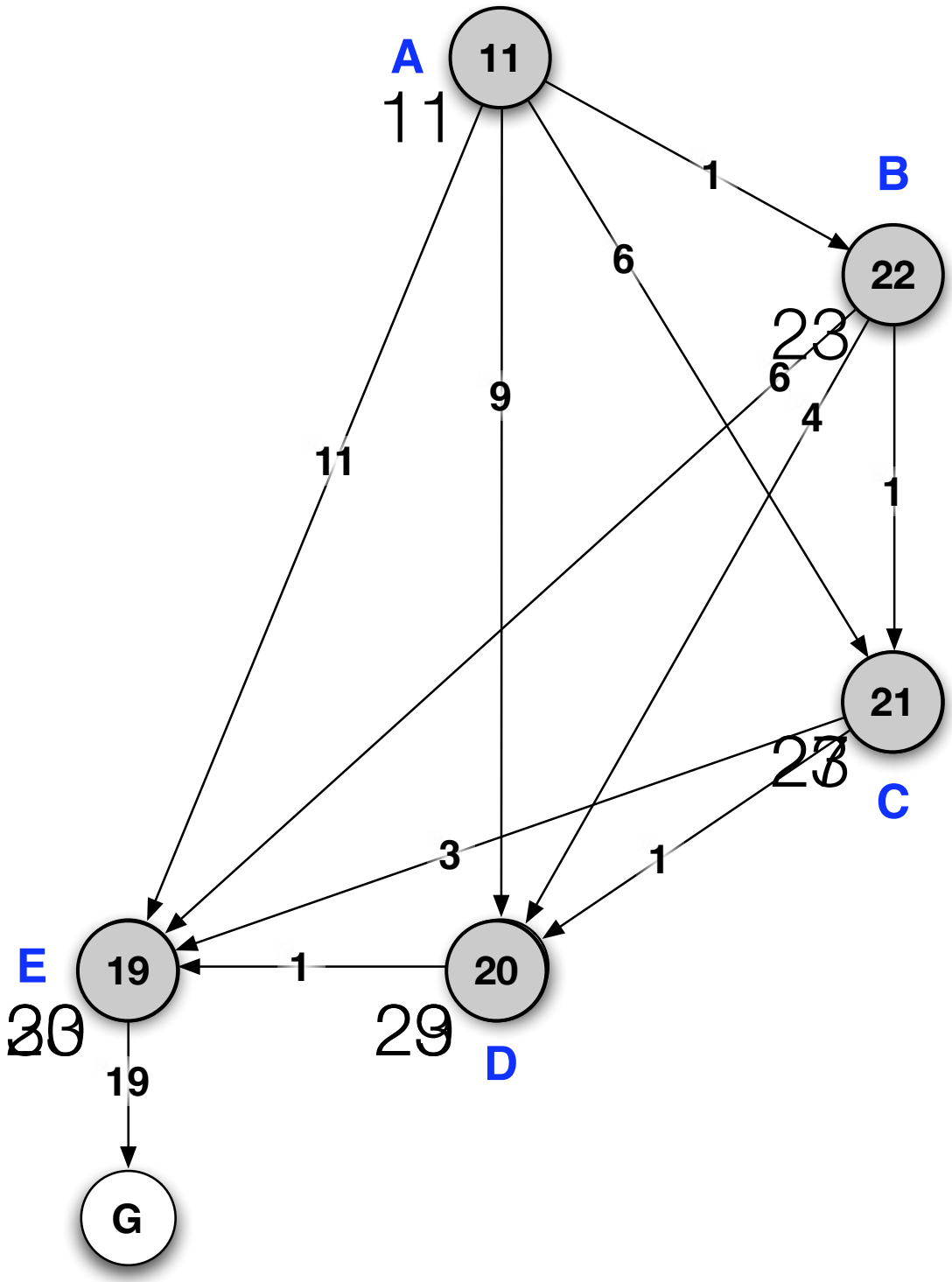
Pathmax rules

- Pathmax:



- BPMX [Felner, et al, 2005]

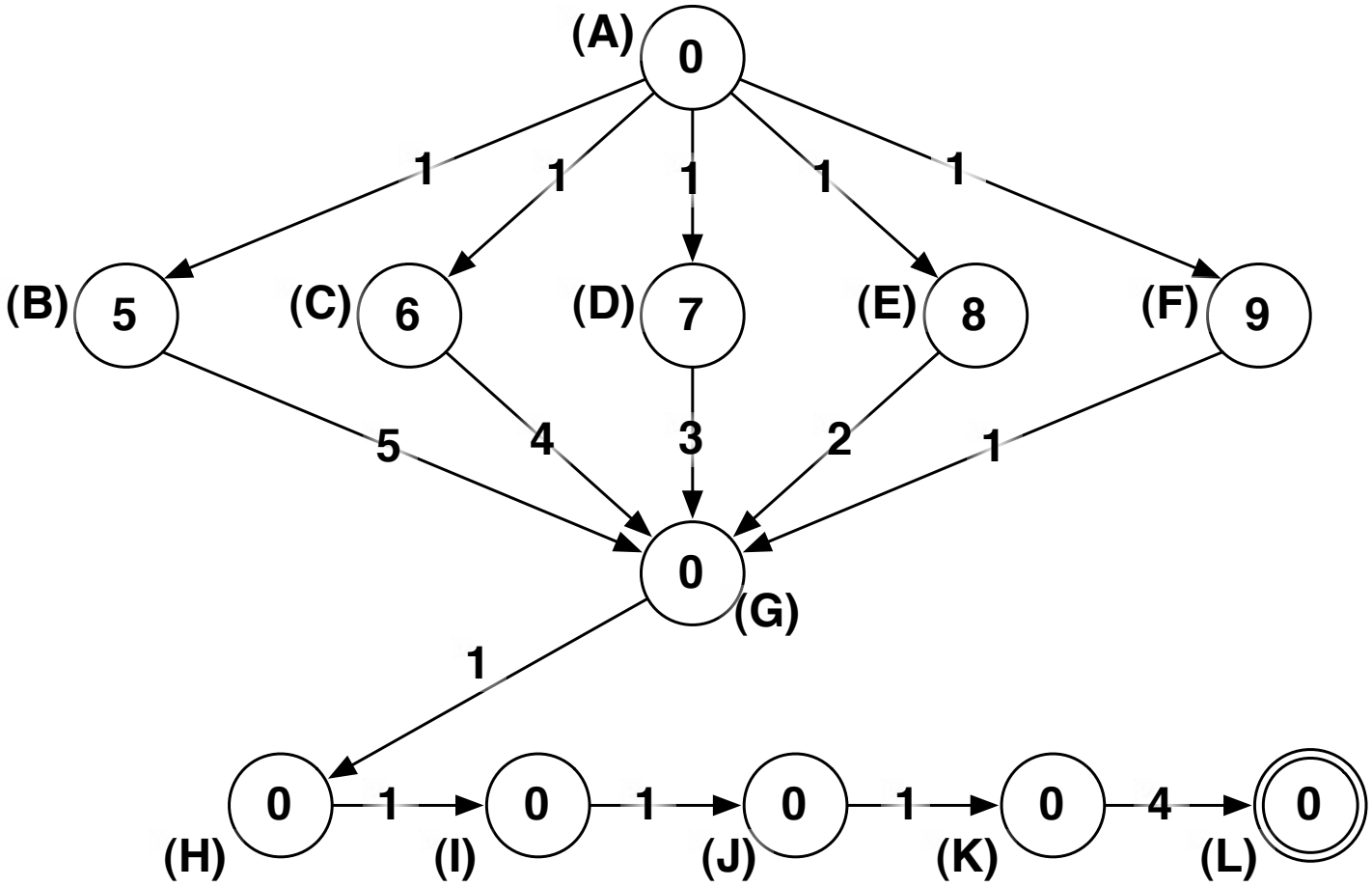




node	f	g	h	F
A	11	0	11	11
E	30	11	19	12
D	29	9	20	13
C	27	6	21	14
B	23	1	22	23
C	23	2	21	23
D	23	3	20	23
E	23	4	19	23
G	23	23	0	23

So...how bad?

- In the worst case, even B' can be $O(N^2)$



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General Inconsistency Bounds

- Suppose A* performs φ(N) > N expansions
  - Some node must be re-opened (φ(N) – N)/N times
    - Pigeon-hole principle
  - If Δ is the minimum change in h-cost for a node
    - h-cost is at least Δ·(φ(N) – N)/N
    - Solution cost is also at least Δ·(φ(N) – N)/N
      - We never open a node with f-cost > solution

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How do we get inconsistency in practice

- Special properties (duality)
- Max of multiple heuristics
  - Too expensive to use all heuristics, so use random subset of heuristics
- Compression

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What is good/bad inconsistency

- "Good" inconsistency
  - There are always good heuristics nearby
  - 1-step BPMX to 'fix' bad values
  - Improve the run-time distribution of h-values
- "Bad" inconsistency
  - Misleading values (worst path has lowest f-cost)
- Note: with no cycles, inconsistency isn't a problem

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BPMX in A*/IDA*

- BPMX is free in IDA*
- More expensive in A*
  - We don't naturally backtrack through closed list
  - Choice:
    - Backup as far as possible
      - $O(N^2)$ cost or unbounded savings
    - Backup only *k* steps O(kN) cost

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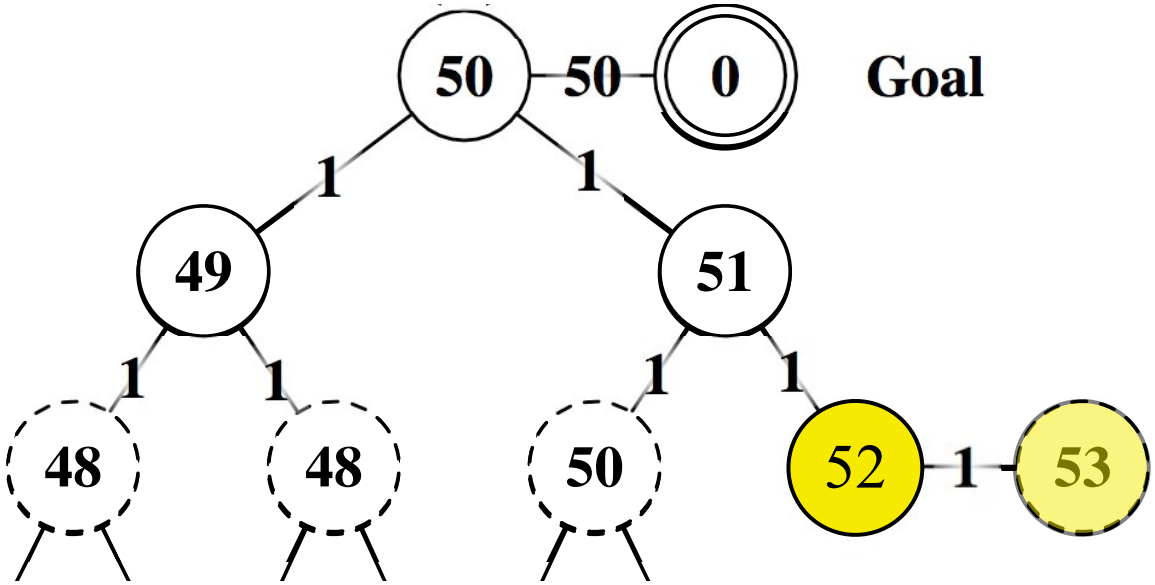
BPMX Best Case



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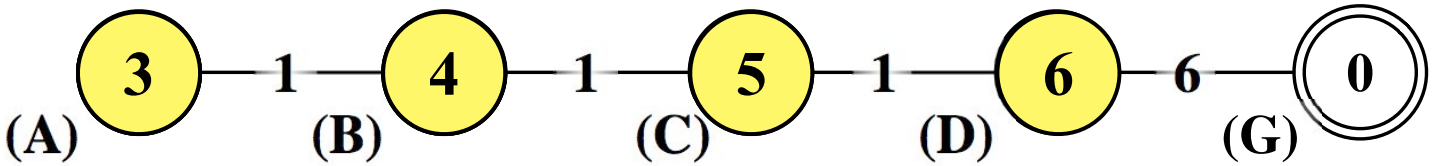
BPMX Worst Case



Abstraction

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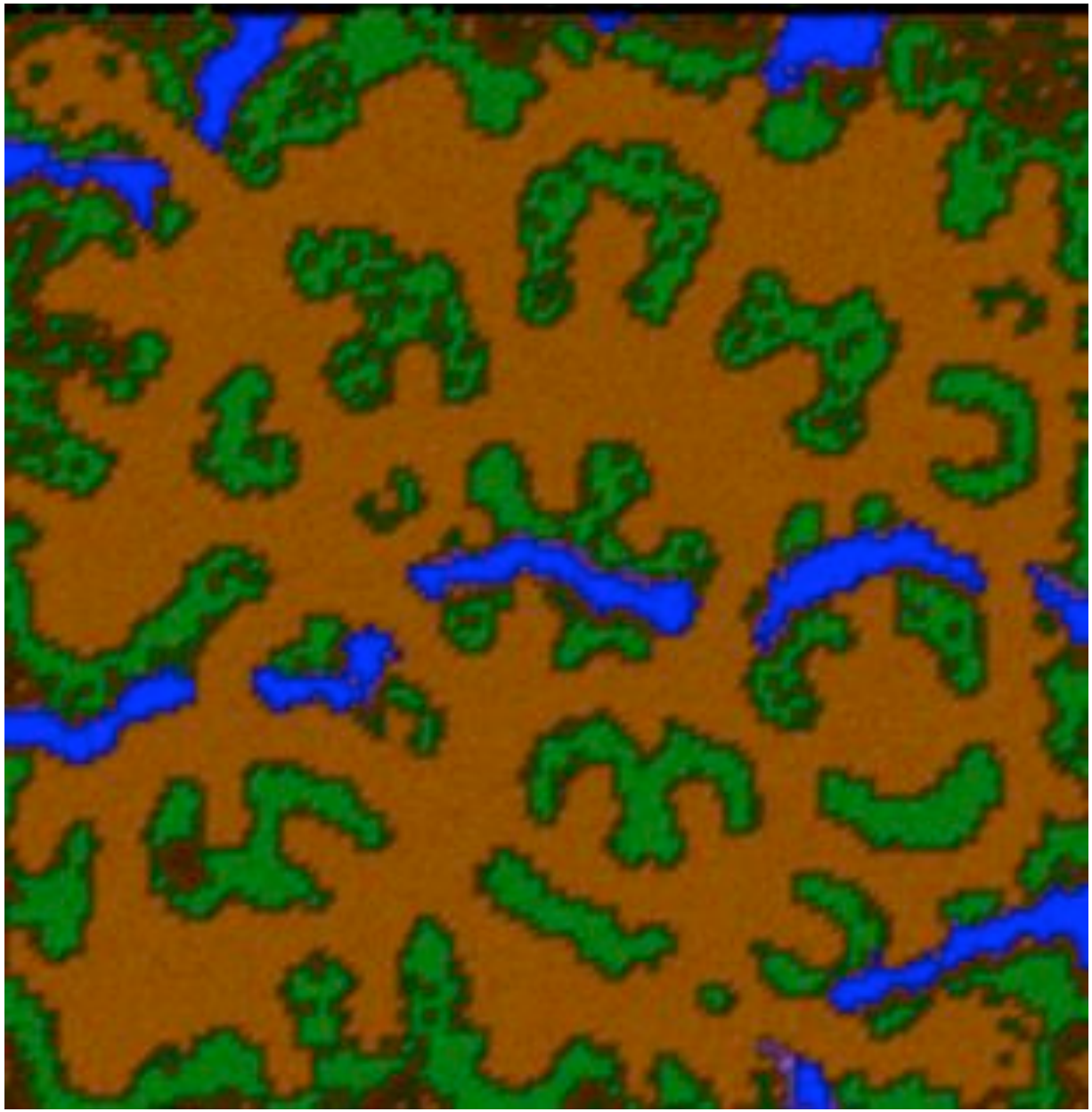
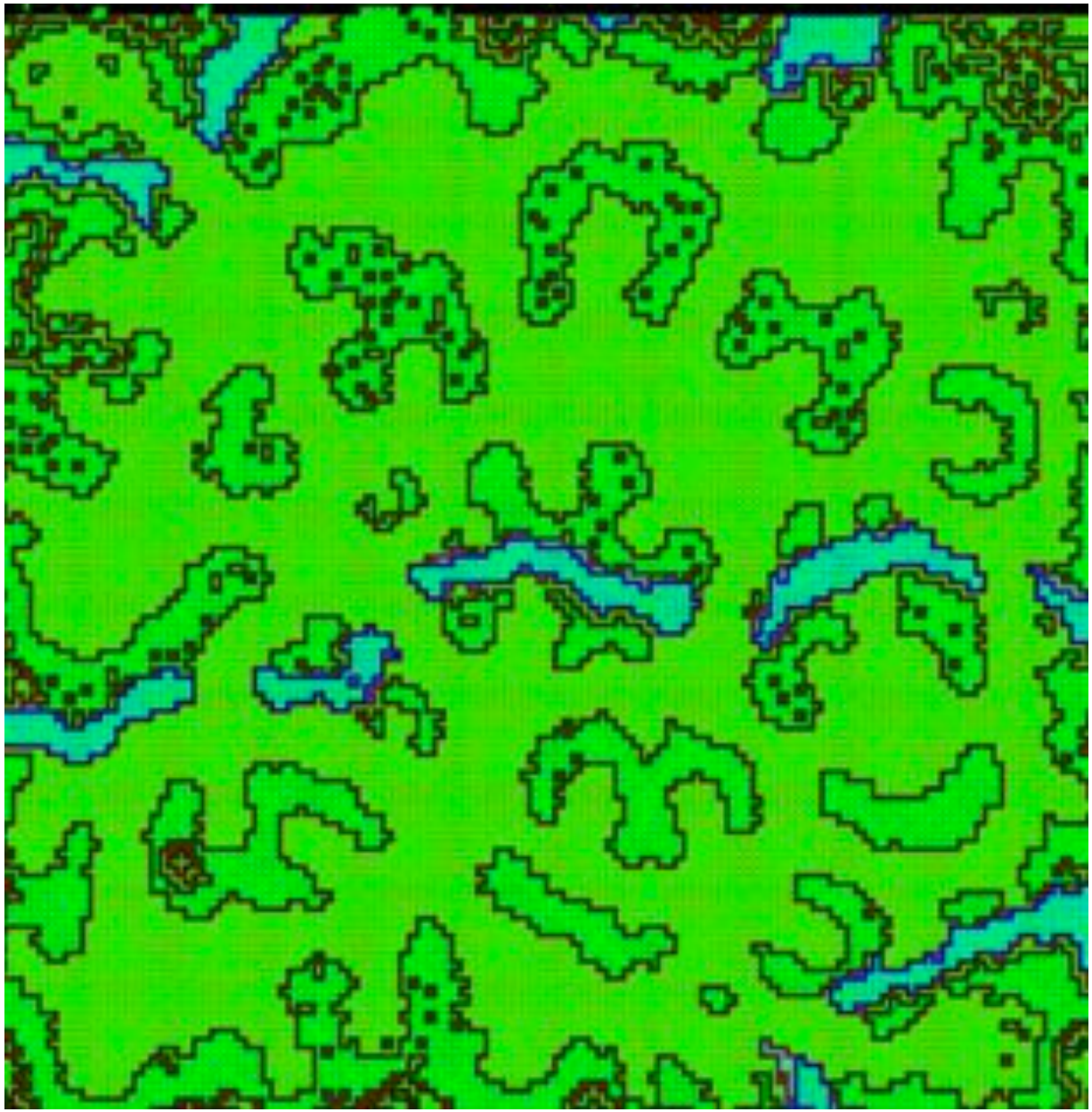
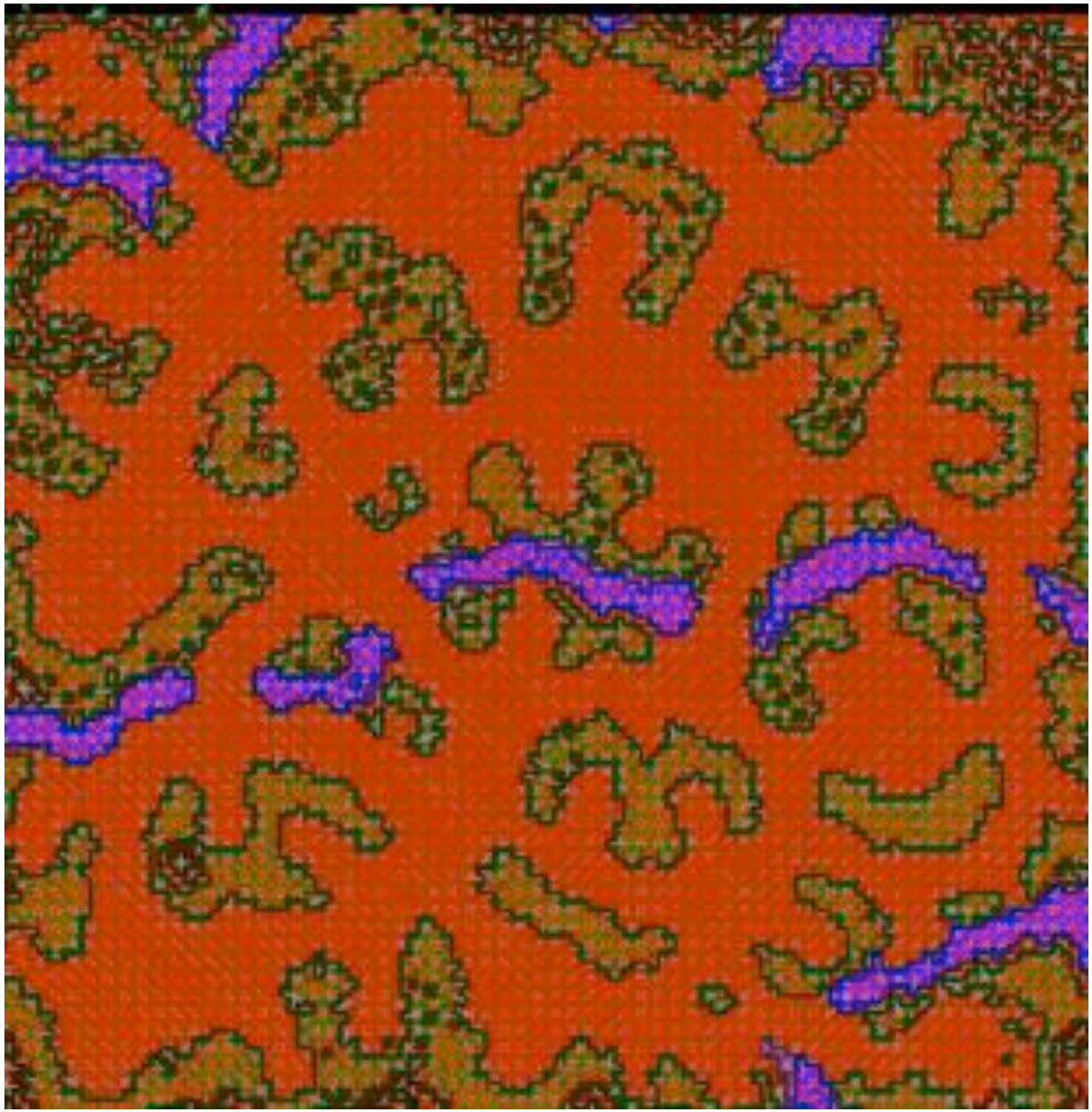
Abstraction and Refinement

- Build a new representation of the state space
  - Want abstraction to be homomorphic (Holte)

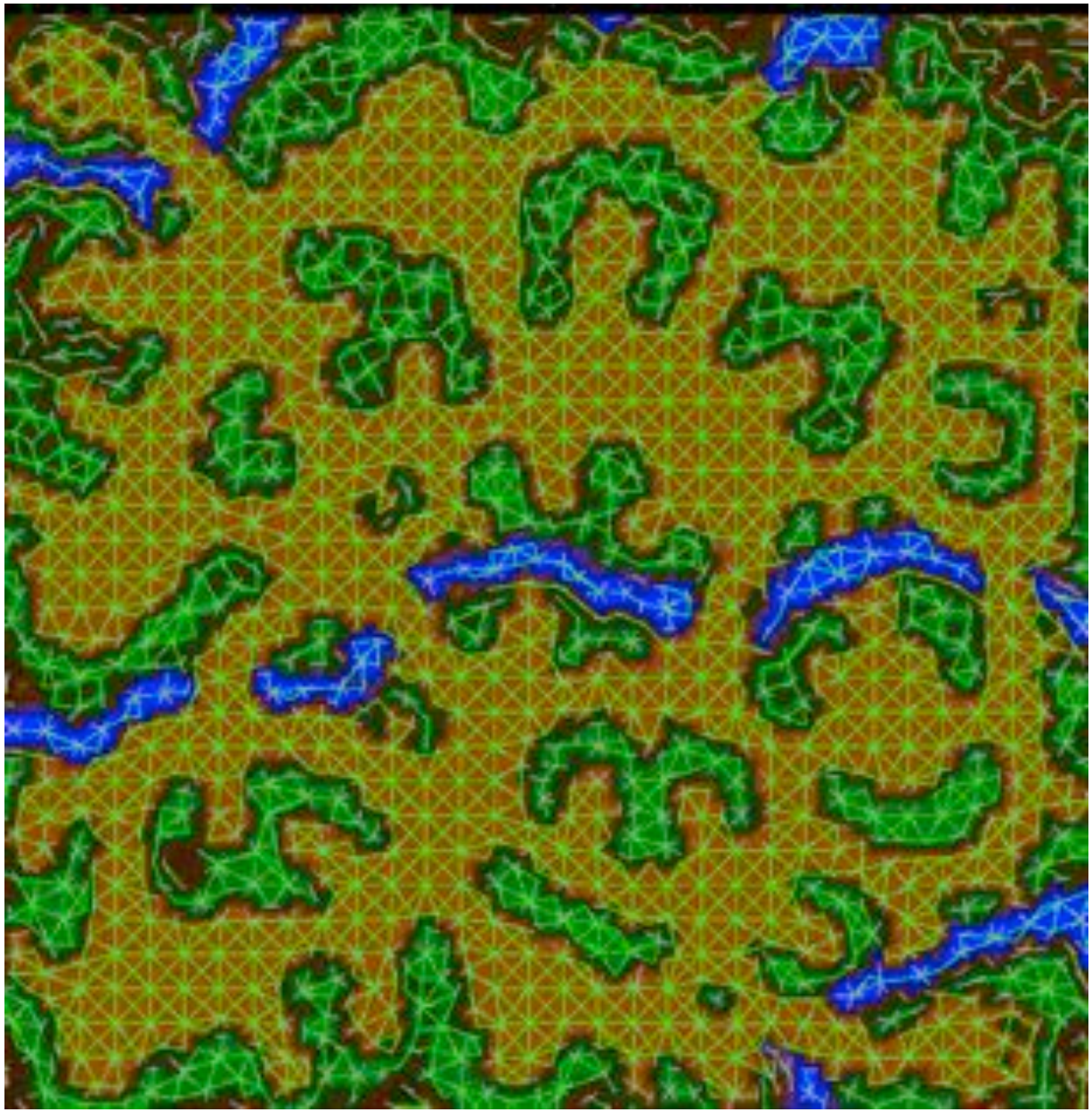




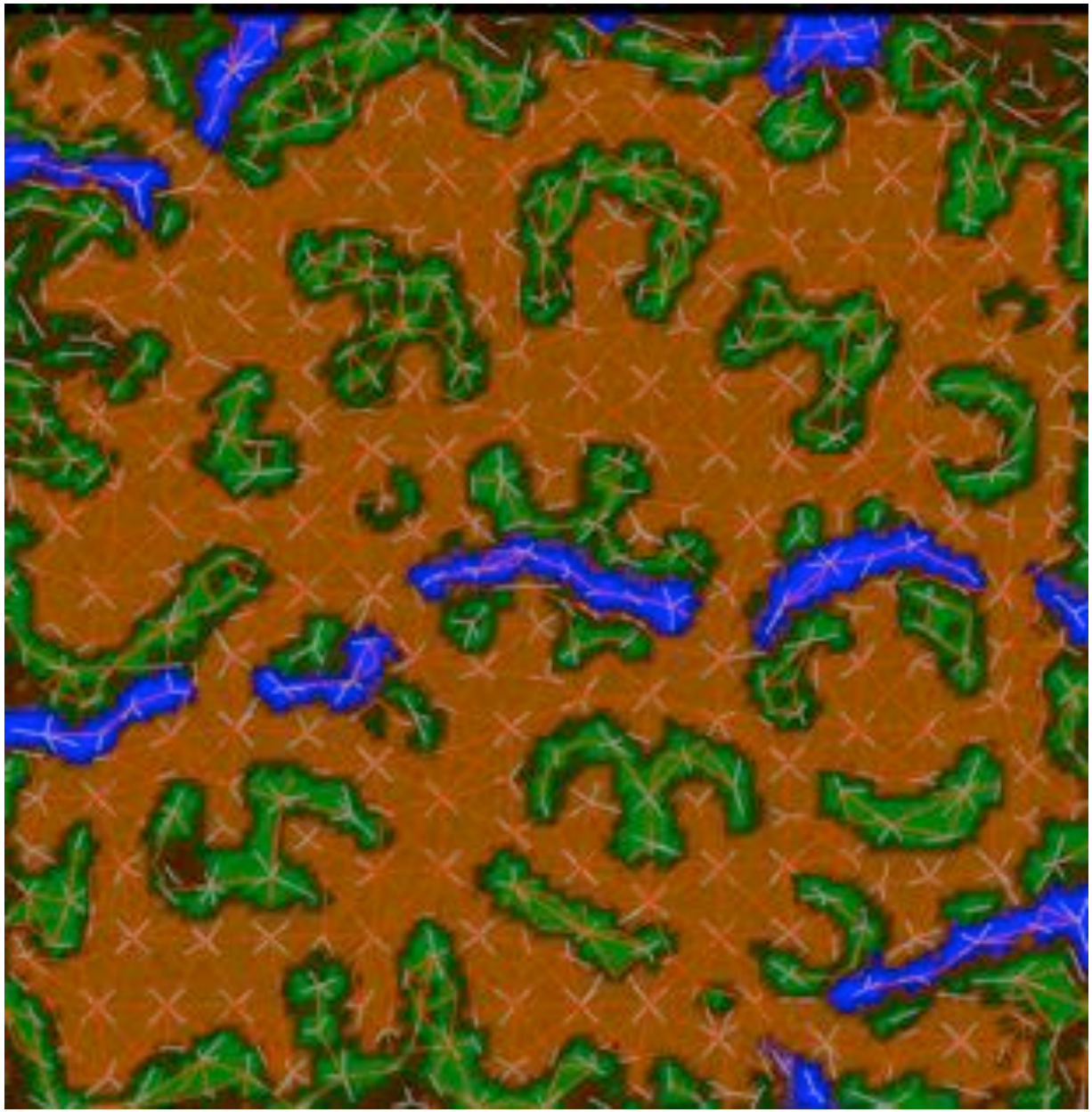
16807 Nodes



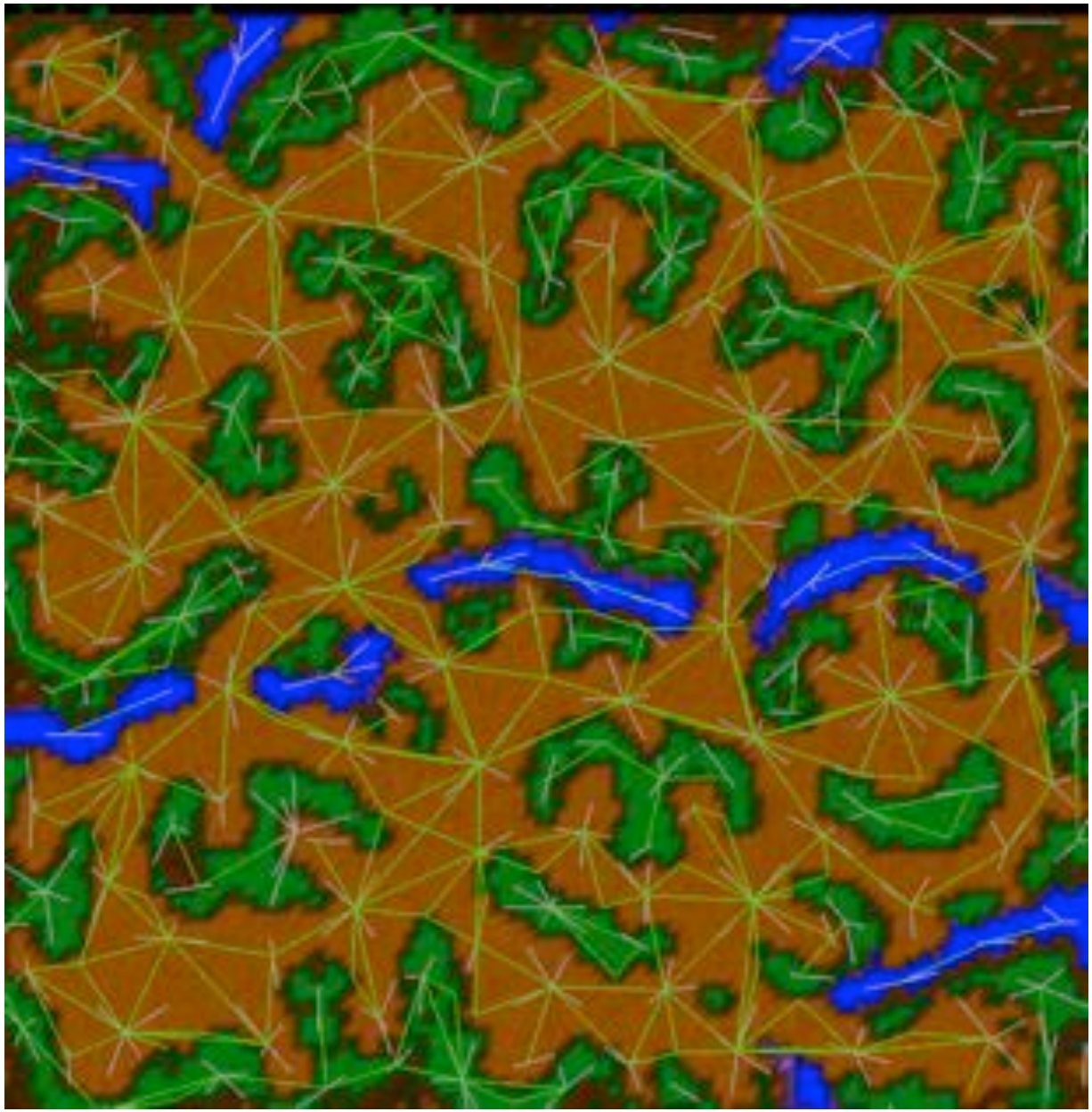
5212 Nodes



1919 Nodes



771 Nodes



316 Nodes

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Algorithm

- Choose group of nodes to abstract together
  - Nodes must be connected
  - Nodes cannot already be abstracted
  - Repeat until all nodes abstracted
- Add an edge between abstract groups if there exists an edge between any two nodes abstracted by each group

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Properties

- Homomorphic abstractions are *refineable*:
  - If any abstract path can be directly refined into a path into the original graph
    - All nodes on refined path abstract into abstract path
  - Every path in the original graph has an abstract counterpart

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Refinement uses

- Used in road networks
- Used in model checking
- Used in heuristic search
- Used in robotics

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Approaches to Refinement-Style Search

- Find abstract path
- Partially refine path
- Refinement corridor

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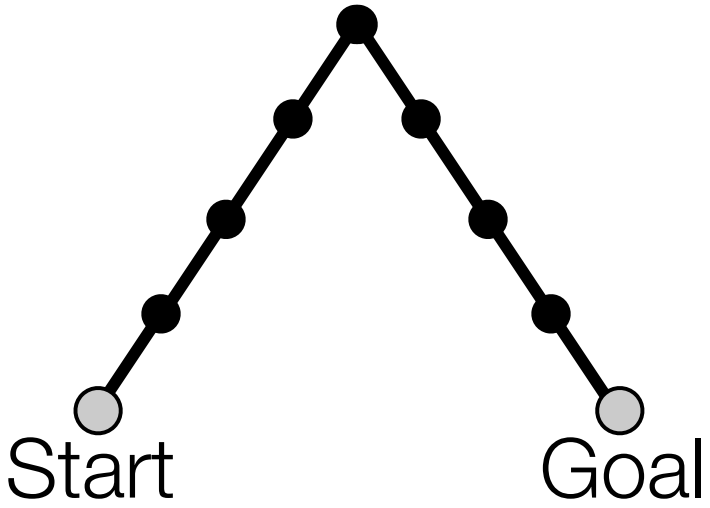
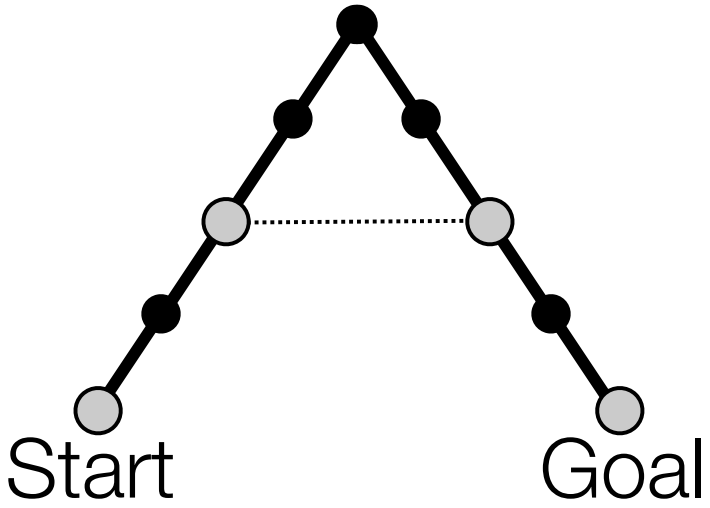
PRA*(k) - Partial-Refinement A*

- Find a complete abstract path
- Until a partial path is available
  - Take first *k* steps of abstract path and refine
- Follow path & refine more steps when needed


Start
Goal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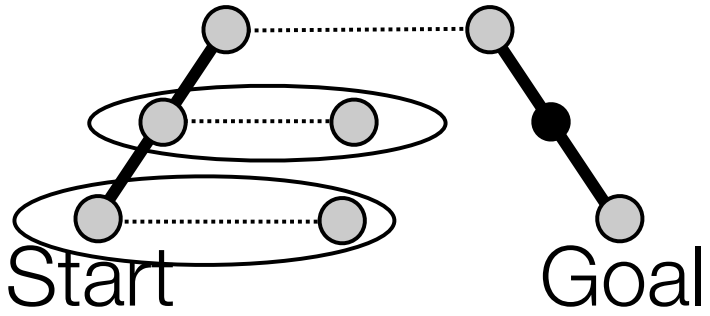
Start
Goal

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Abstraction in Road Networks

- Road networks happen to have special properties

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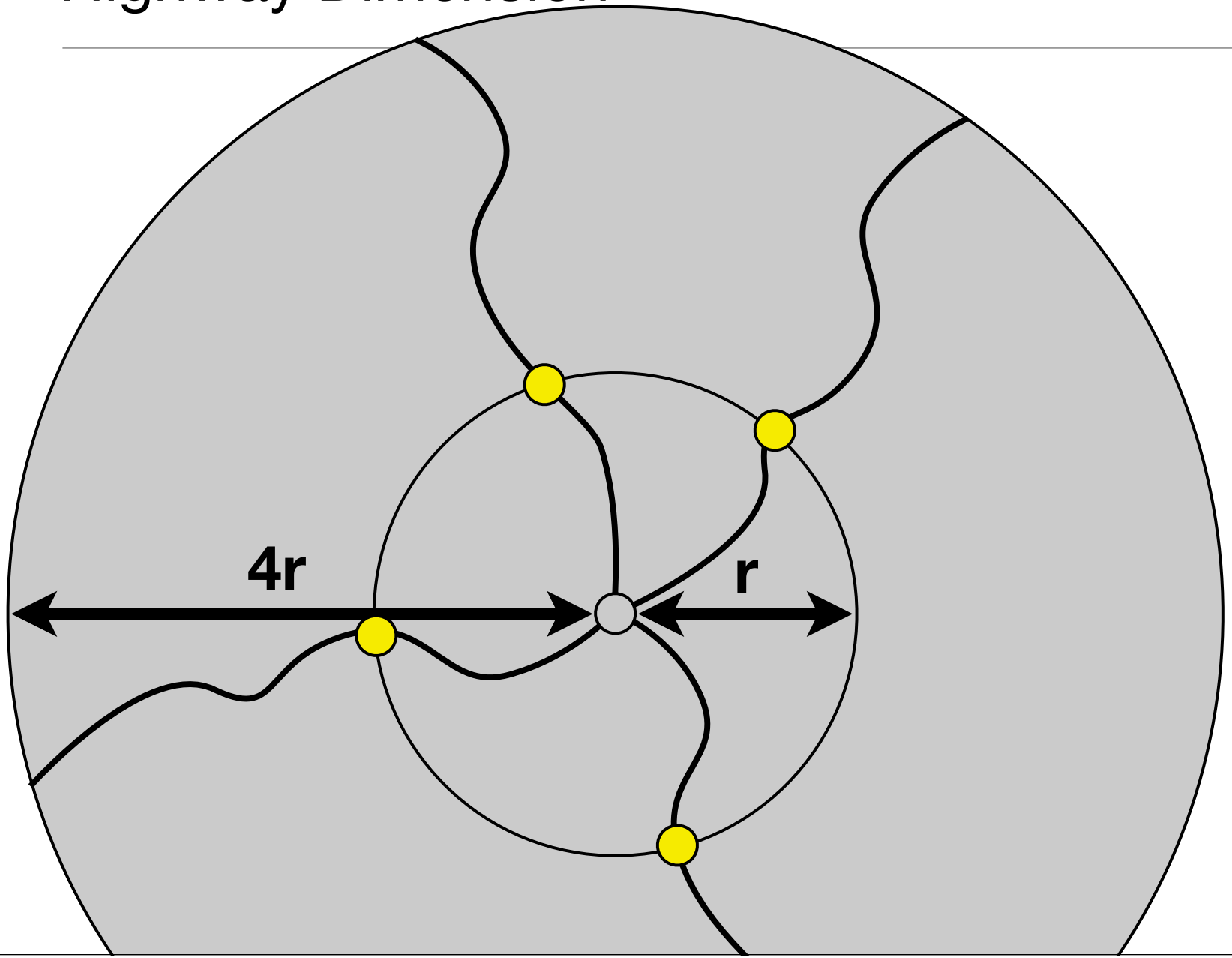
Highway Dimension



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Highway Dimension

- Problems with low highway dimension can be solved quickly
  - Abraham et. al., 2010
- Suite of techniques developed
  - Reach (Goldenberg, et al)
  - Contraction hierarchies (Geisberger, et al)
  - Transit node routing (Bast, et al)

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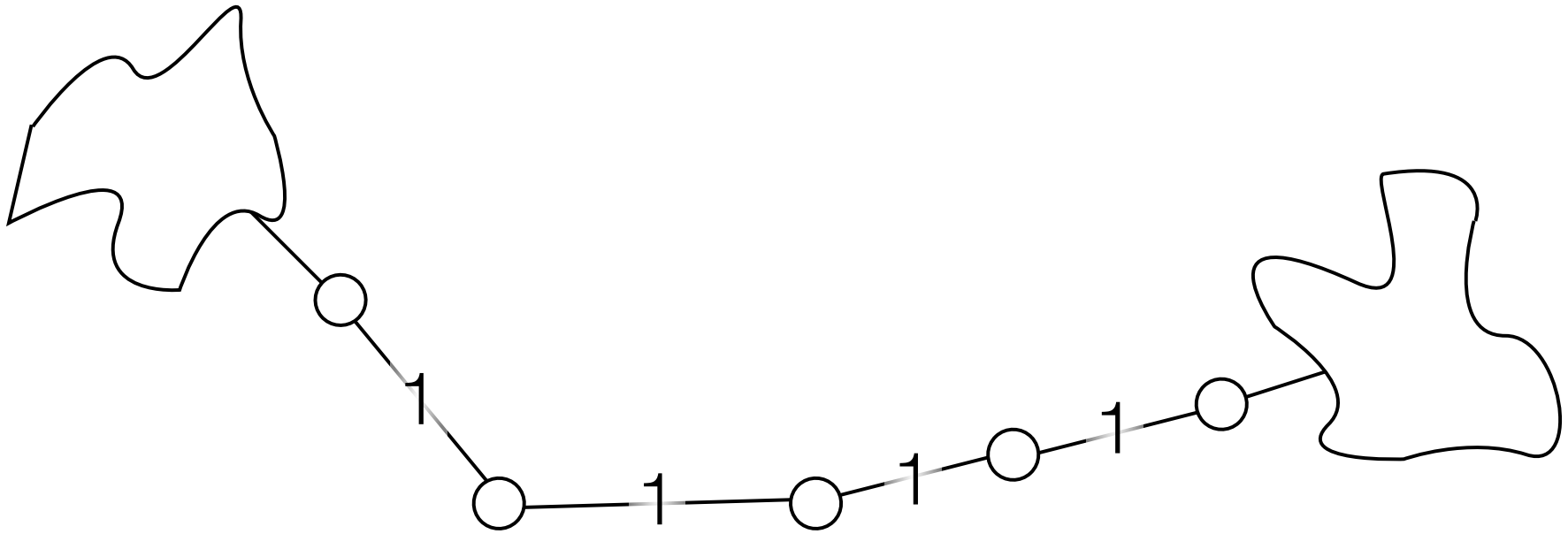
Contraction Hierarchies



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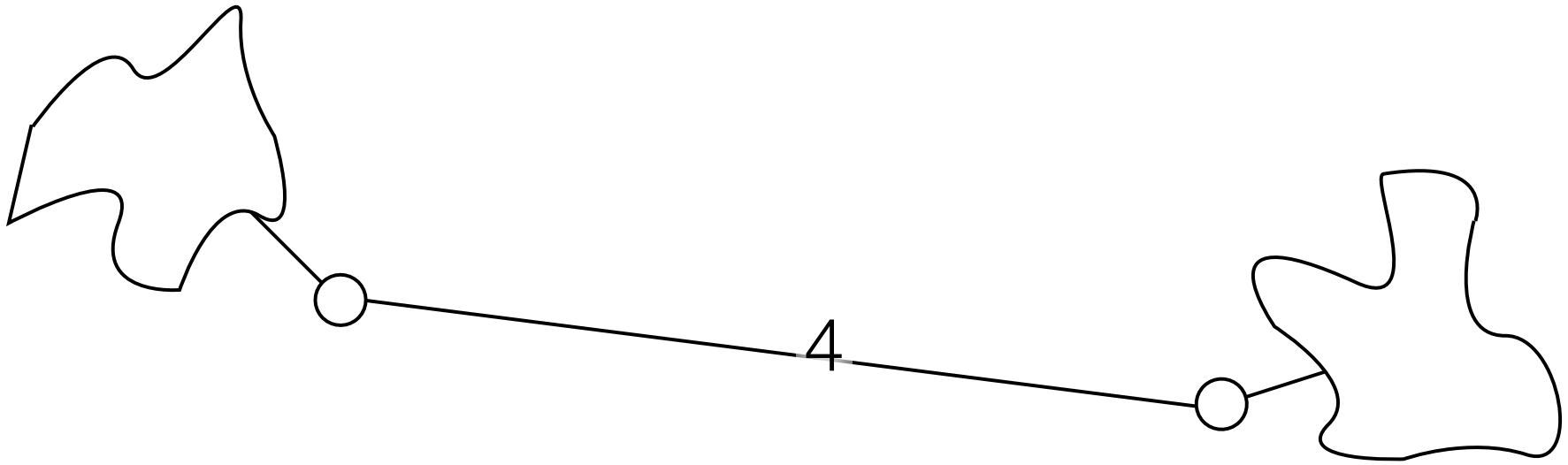
Initial Motivation: Lines



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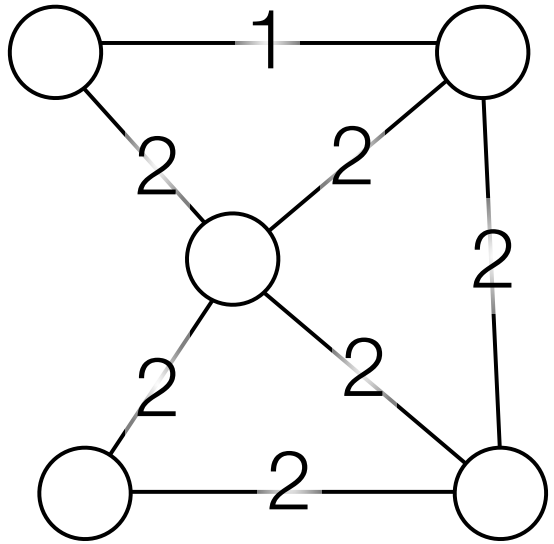
Generalization



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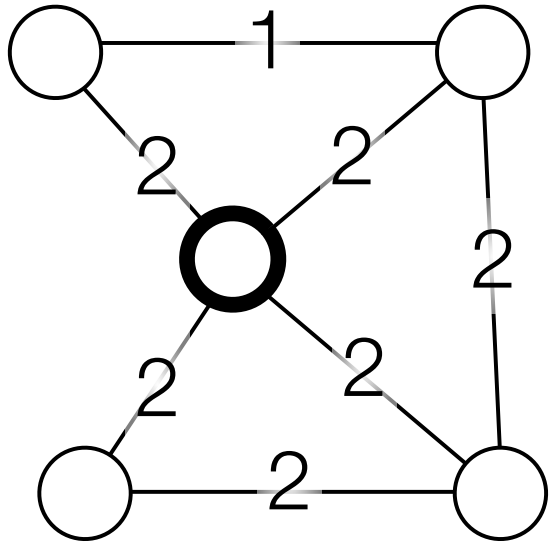
Generalization



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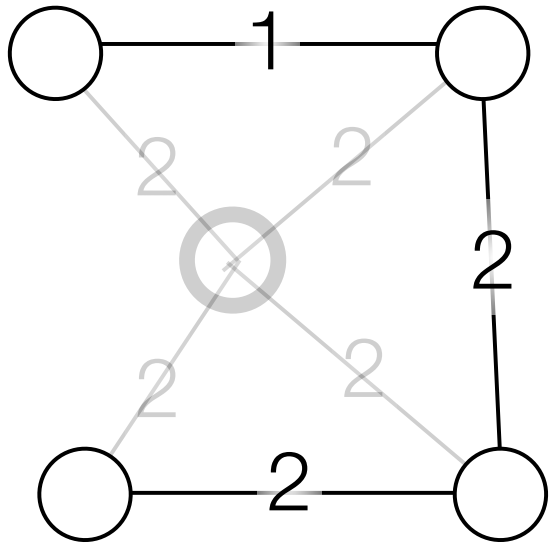

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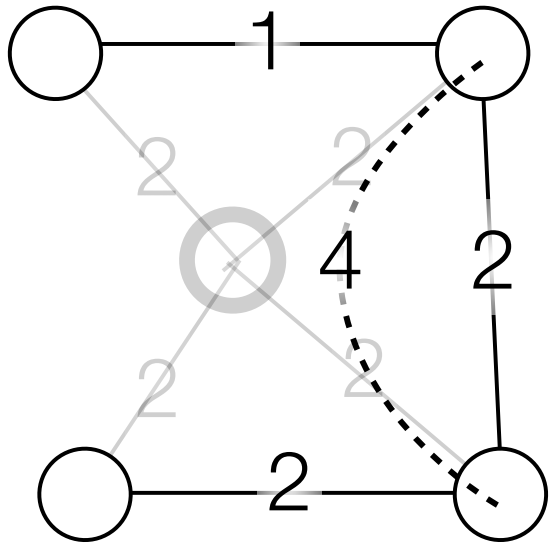
Generalization



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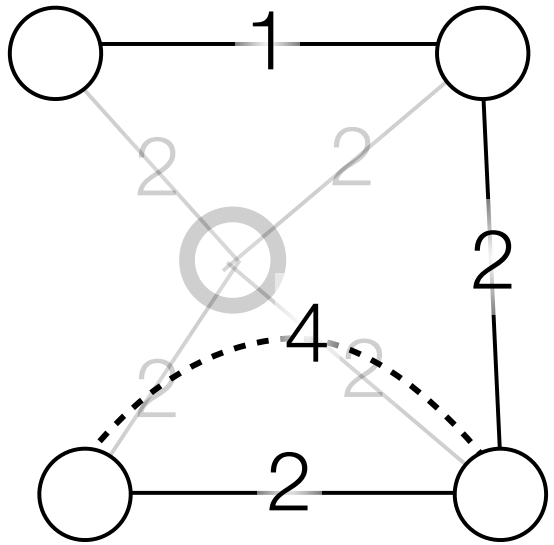
Generalization



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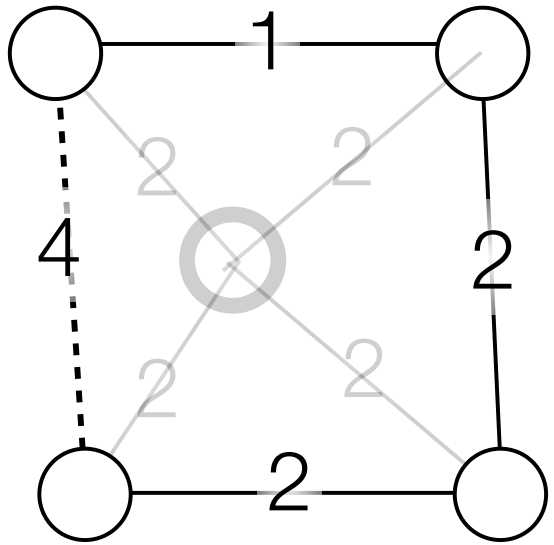
Generalization



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Building Contraction Hierarchies

- Choose most important node *n*
  - For all pairs of neighbors, check if removing *n* influences the shortest path between neighbors
    - If not, just remove *n*
    - If so, add shortcut edge with the same cost as the shortest path through *n*

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Node Importance

- Ad-hoc ordering for nodes:
  - **Edge difference**: how many edges are removed/ introduced when *n* is contracted
  - **Original edges**: how many edges have already been abstracted below shortcut edges introduced when *n* is contracted
  - **Upward path length**: max of unpacked path length
  - **Contracted neighbors**: how many neighbors are already contracted

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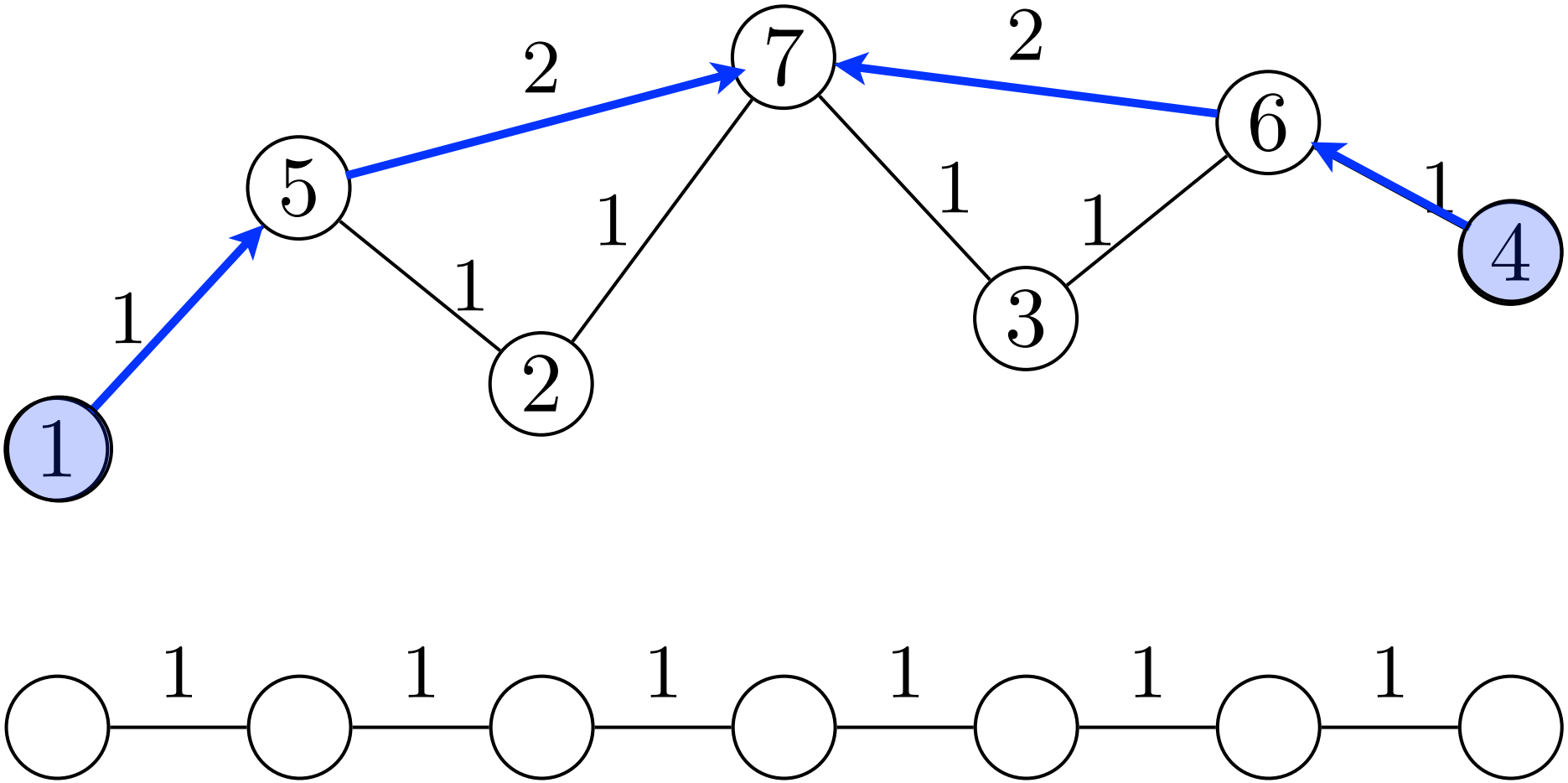
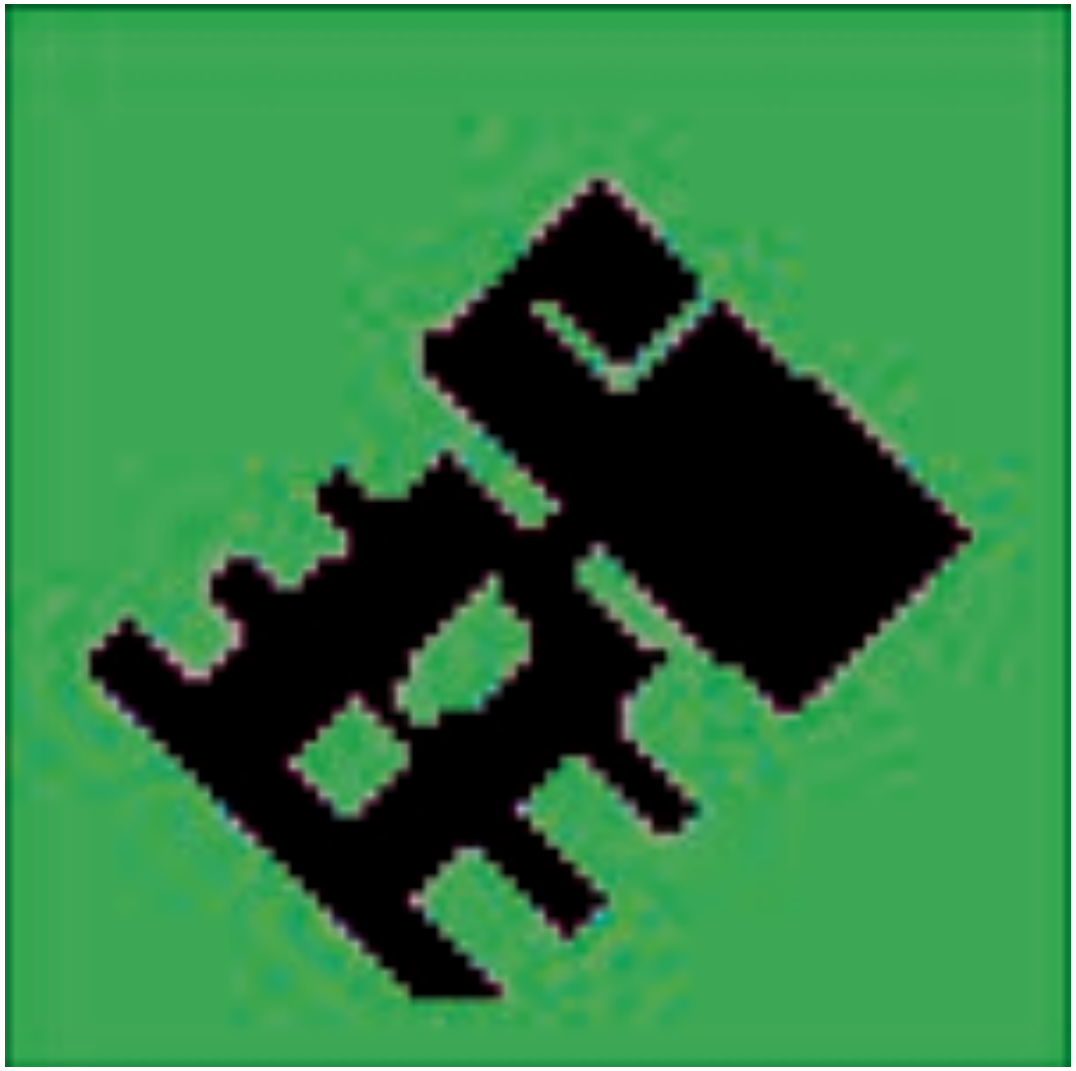
Using CHs



Easy for Abstraction
Harder for CH



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Easy for CH
Harder for Abstraction

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Use in practice

- In practice:
  - Do not contract the whole graph
  - Contract until the graph reaches a particular size
  - Then search with differential heuristics

Deployment:
Dragon Age Origins

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Example: Dragon Age (BioWare)

- 2006 - BioWare approached UofA
  - Pathfinding using 100ms (1ms available)
  - No memory budget
- Initial implementation (late 2006 - early 2007)
- Additional enhancements (2008)
- Achieved 100μs average per unit per frame
  - Computation spread across frames

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Pathfinding Architecture

High-level planning	Sparse graph
Medium-level planning	Sparse graph
Low-level planning	Grid
Executed path	Arbitrary line segments

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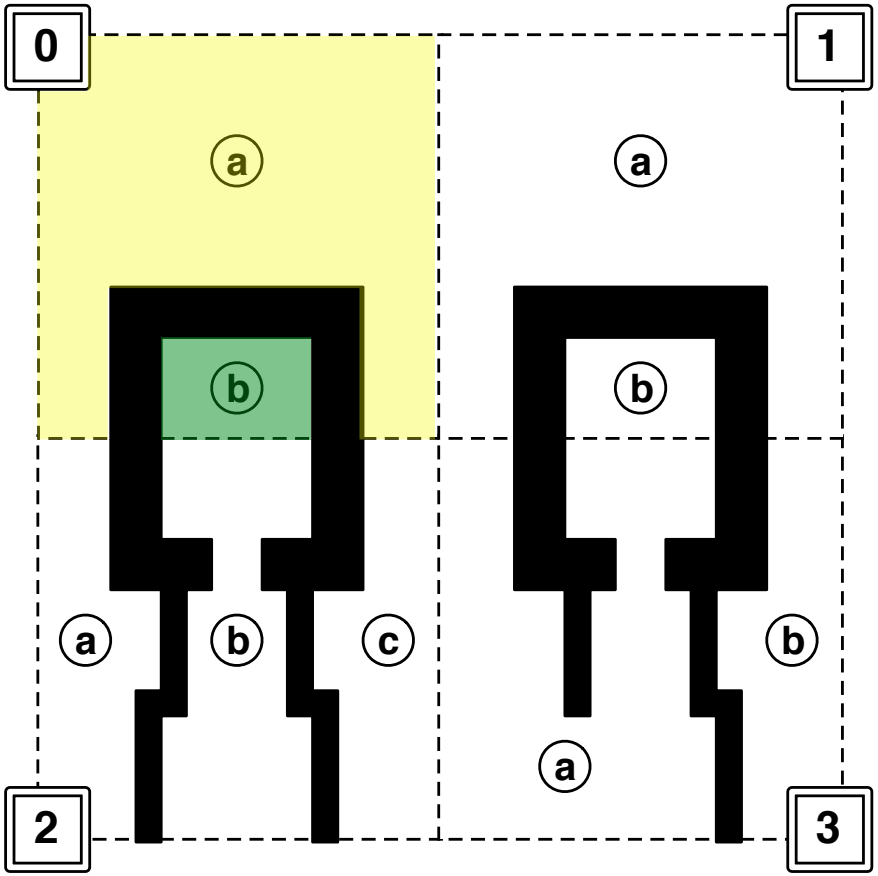
Dragon Age - Additional Enhancements

- Many new features over previous games:
  - Map-wide pathfinding
  - Trap/area effect avoidance
  - Improved static obstacle/creature avoidance
  - Improved path-following animation
  - Many other small performance tweaks

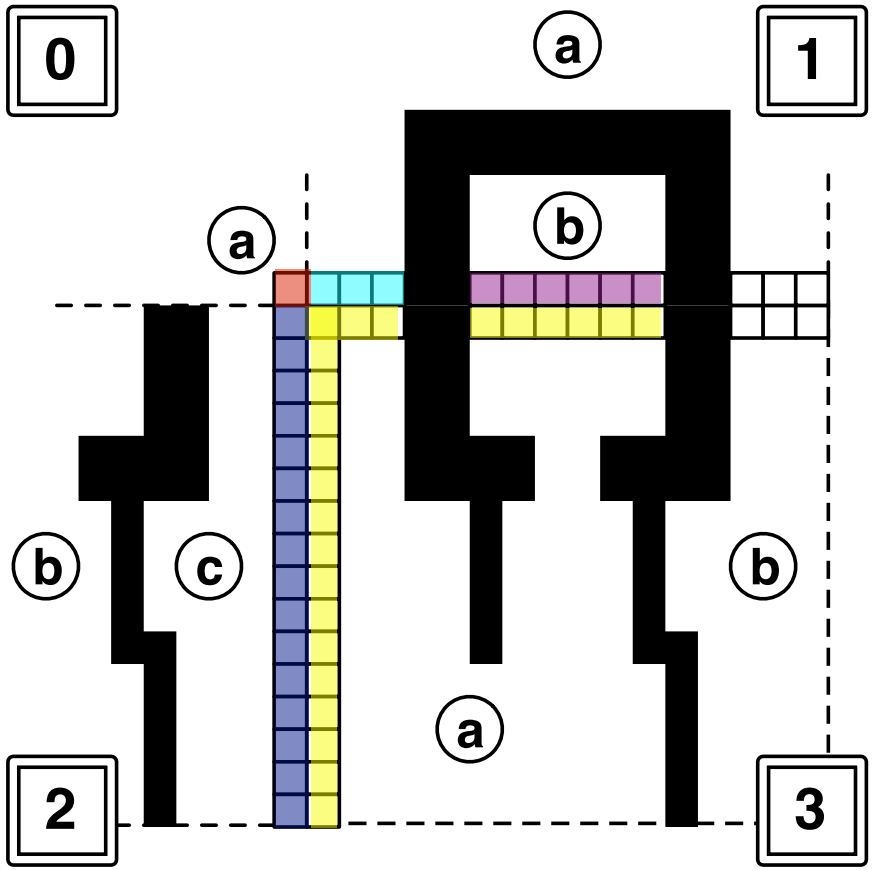
Sectors / Regions

- Divide world into large sectors
  - Fixed size
  - Index implicitly
- Divide sectors into regions
  - Regions entirely connected
  - Regions have a *center* point



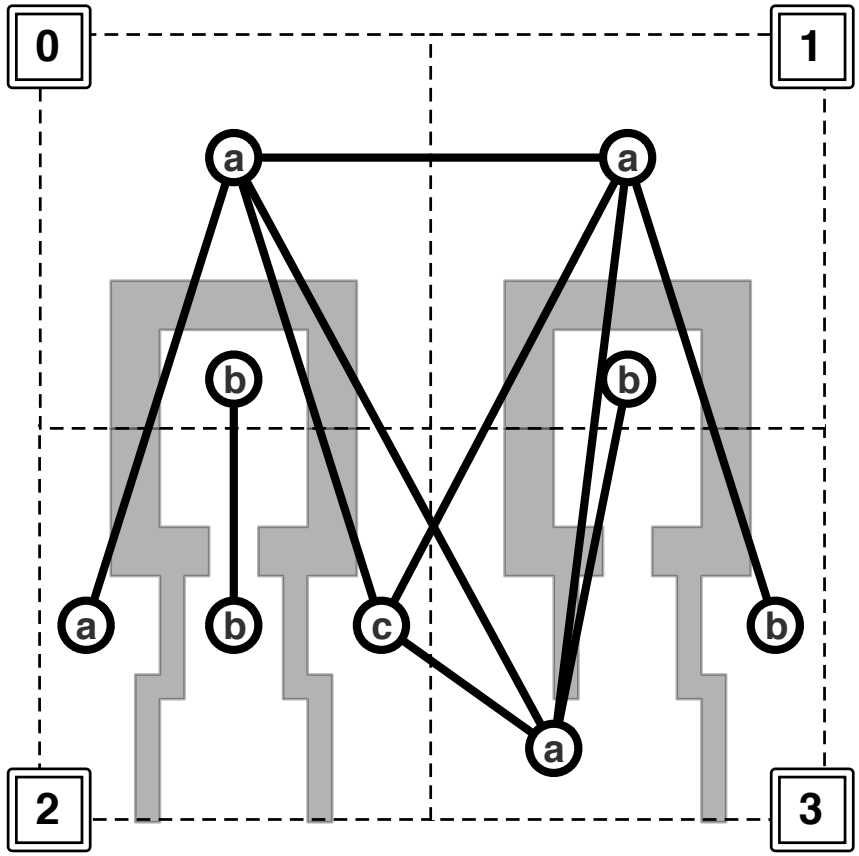
Edges

- Look at borders of regions to determine edges



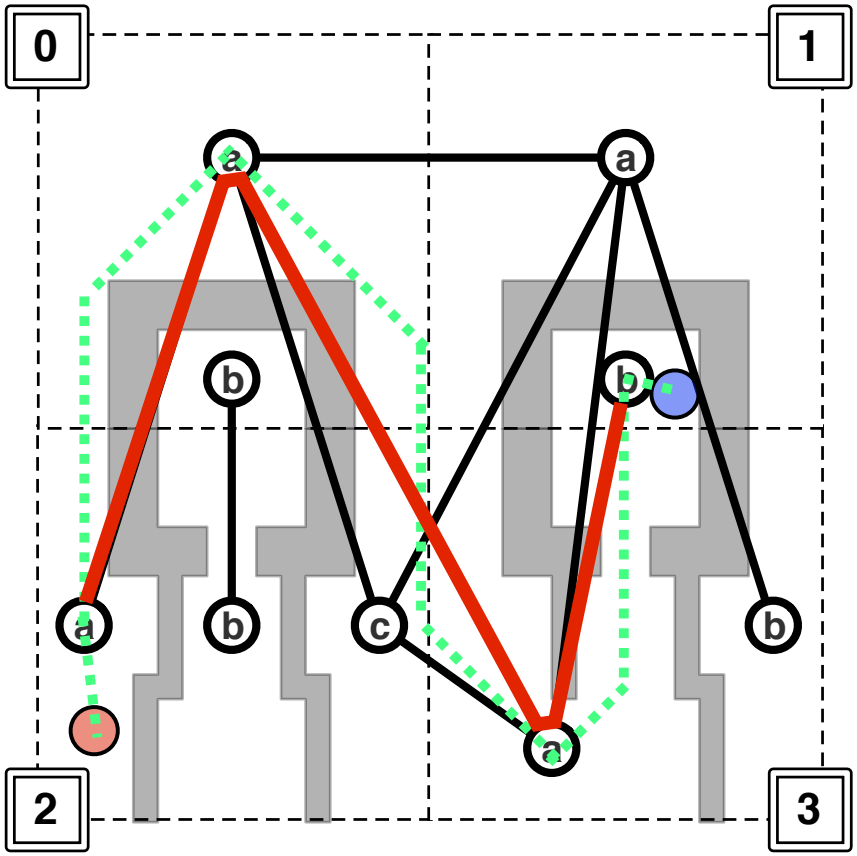
Abstract Graph

- Original Map:
  - 32x32 = 1024 cells
- Abstract Graph:
  - 9 nodes
  - 10 edges



Usage Example

- Find abstract parents
- Find abstract path
- Find real path



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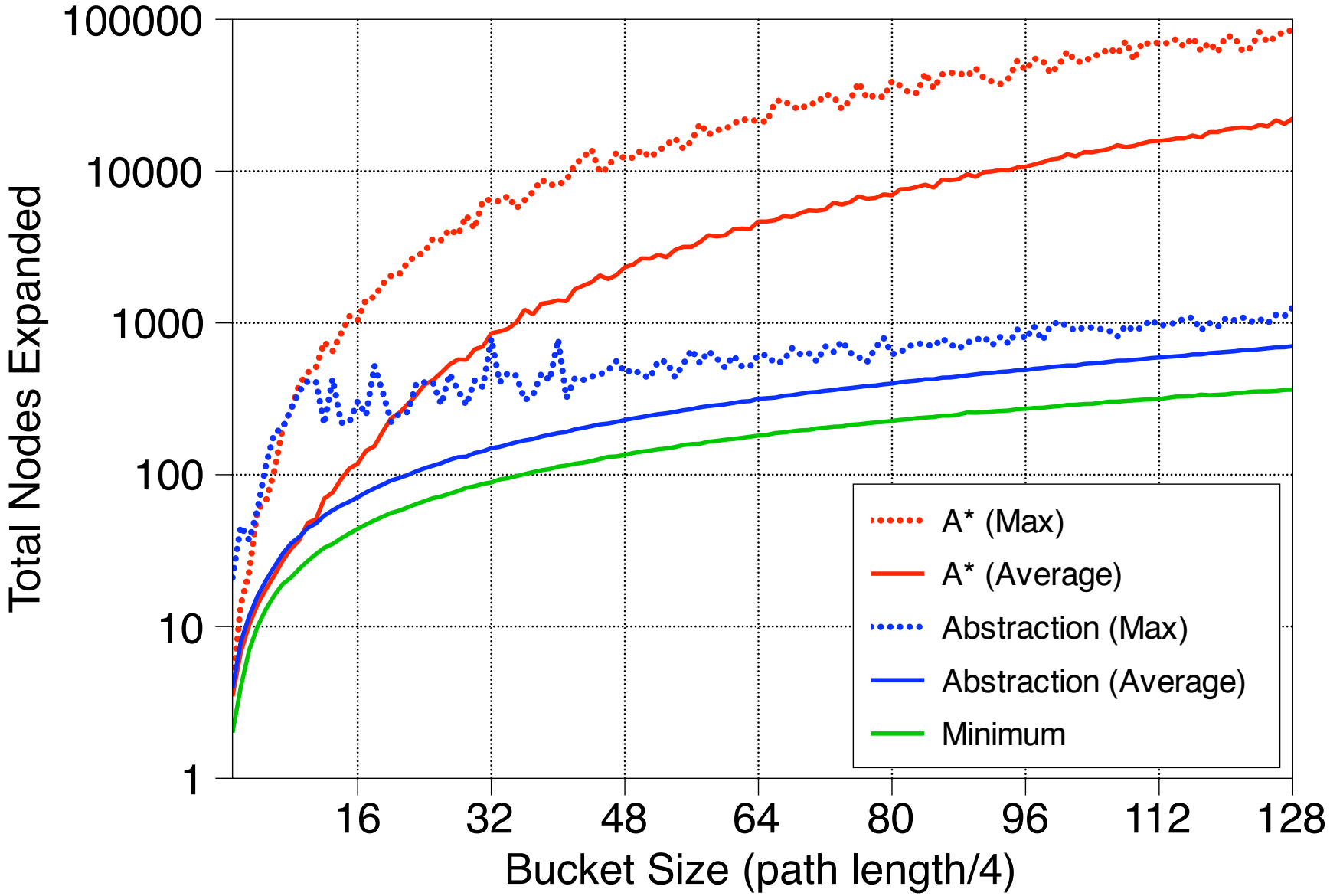
Total Work Compared to A*



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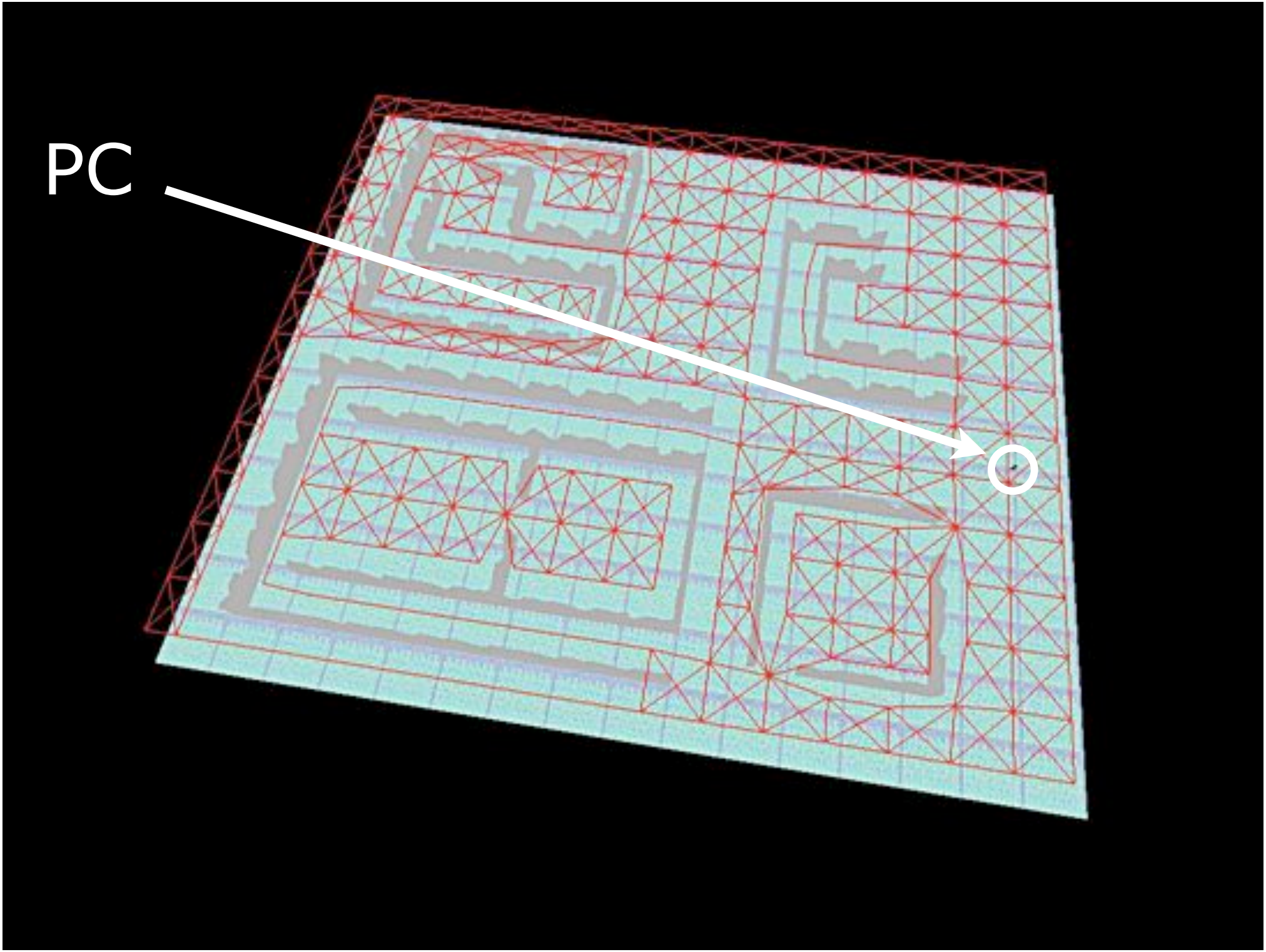
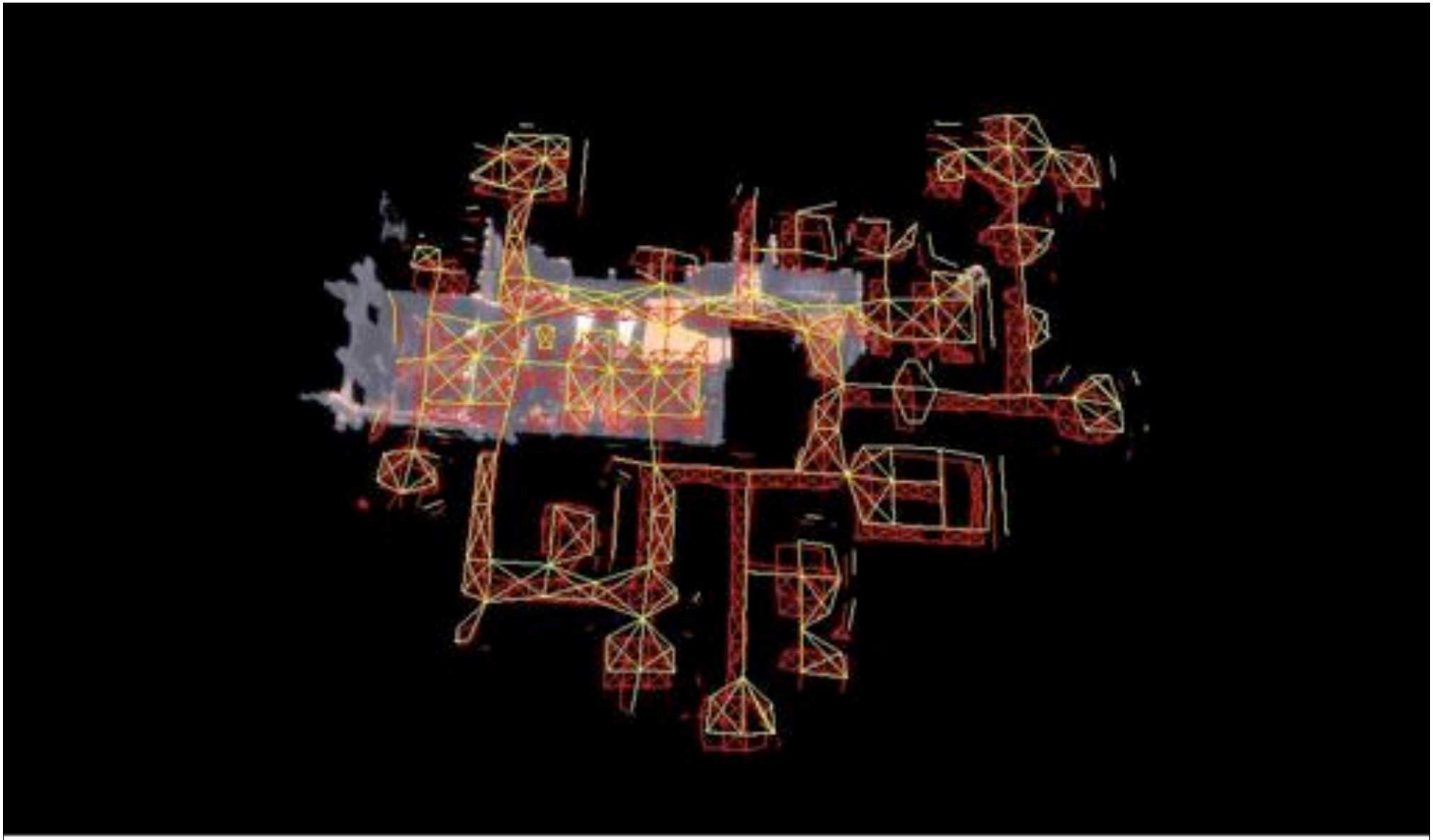
Dragon Age: Origins

Engine Features


PC

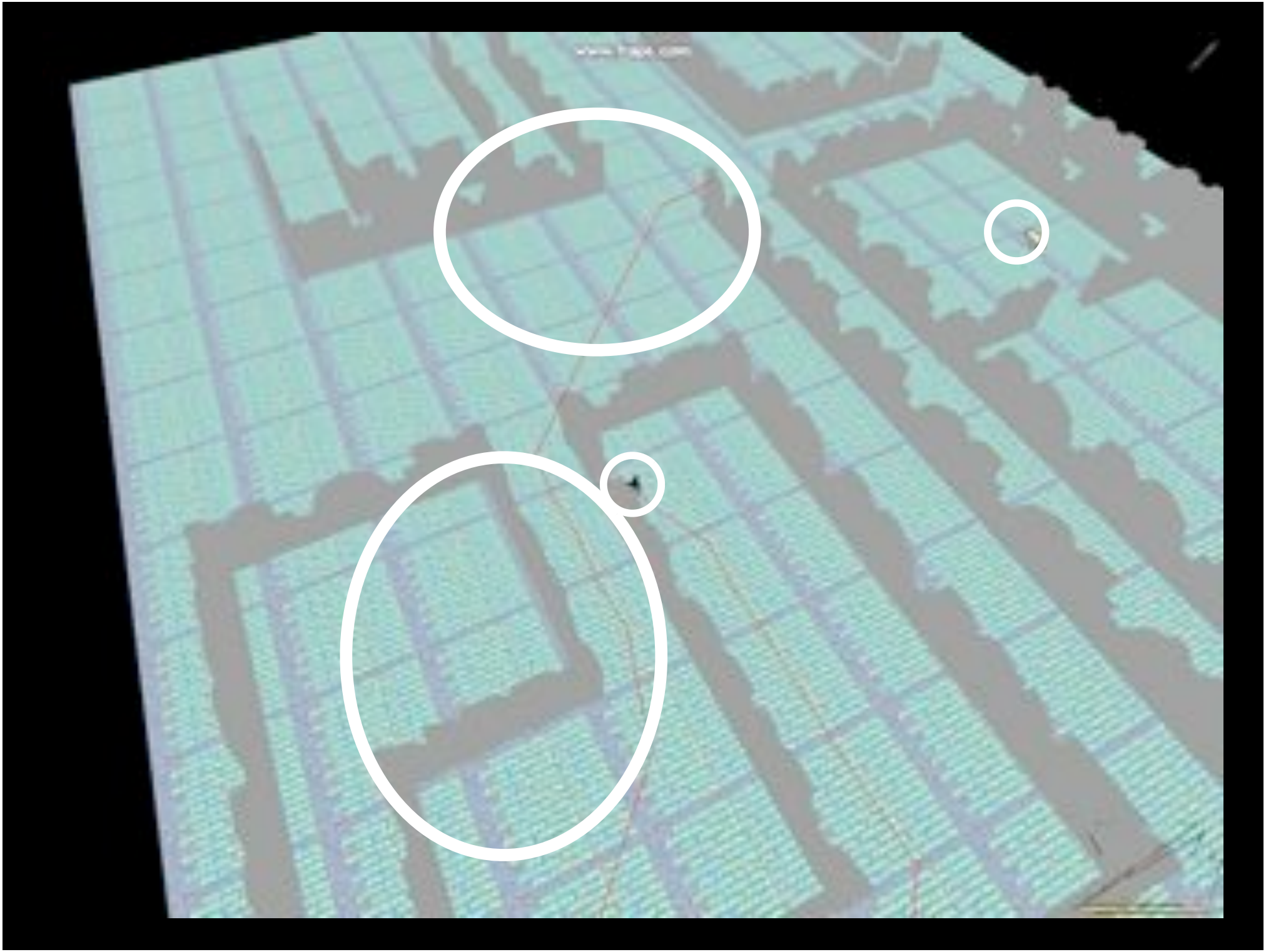


Dragon Age Pathfinding

Top Two Layers of Abstraction

Dragon Age Pathfinding

Outdoor Map

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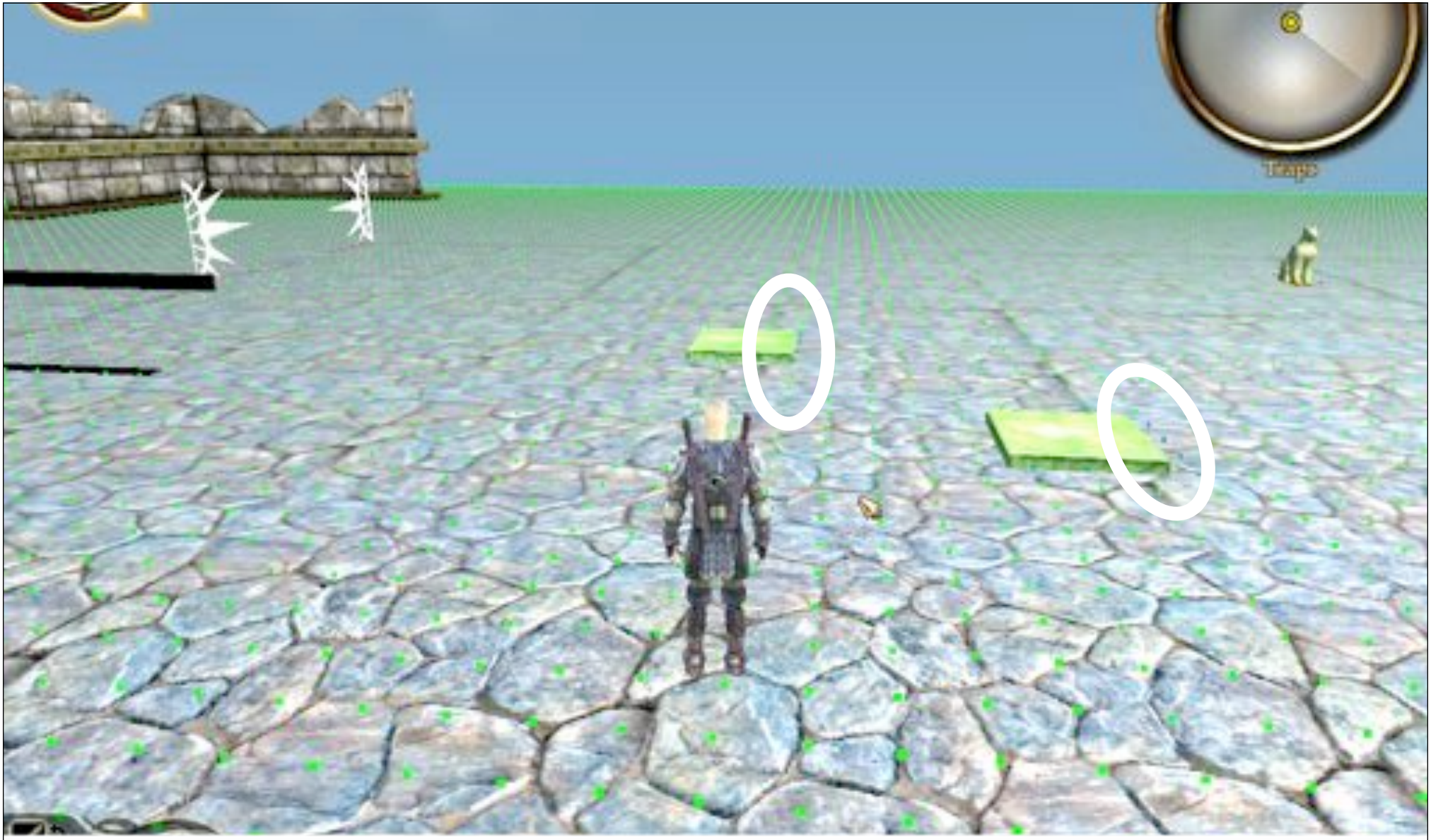

Avoiding a trap


Camp

Artifacts of Abstraction

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Errors at the grid level

- Grid versus real-valued locations
  - PC/NPC location is a real-valued location
  - Mouse clicks are real-value location
  - Low-level planning is on a grid


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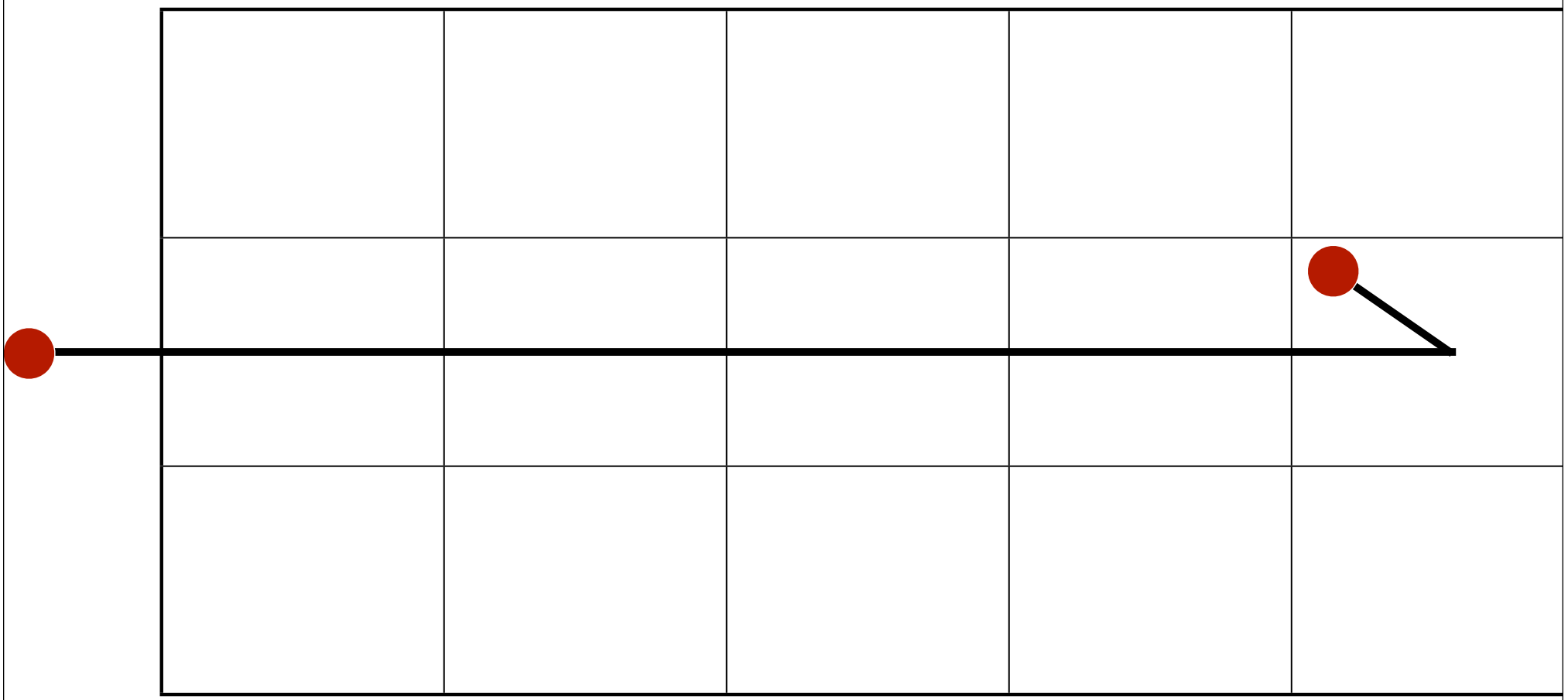
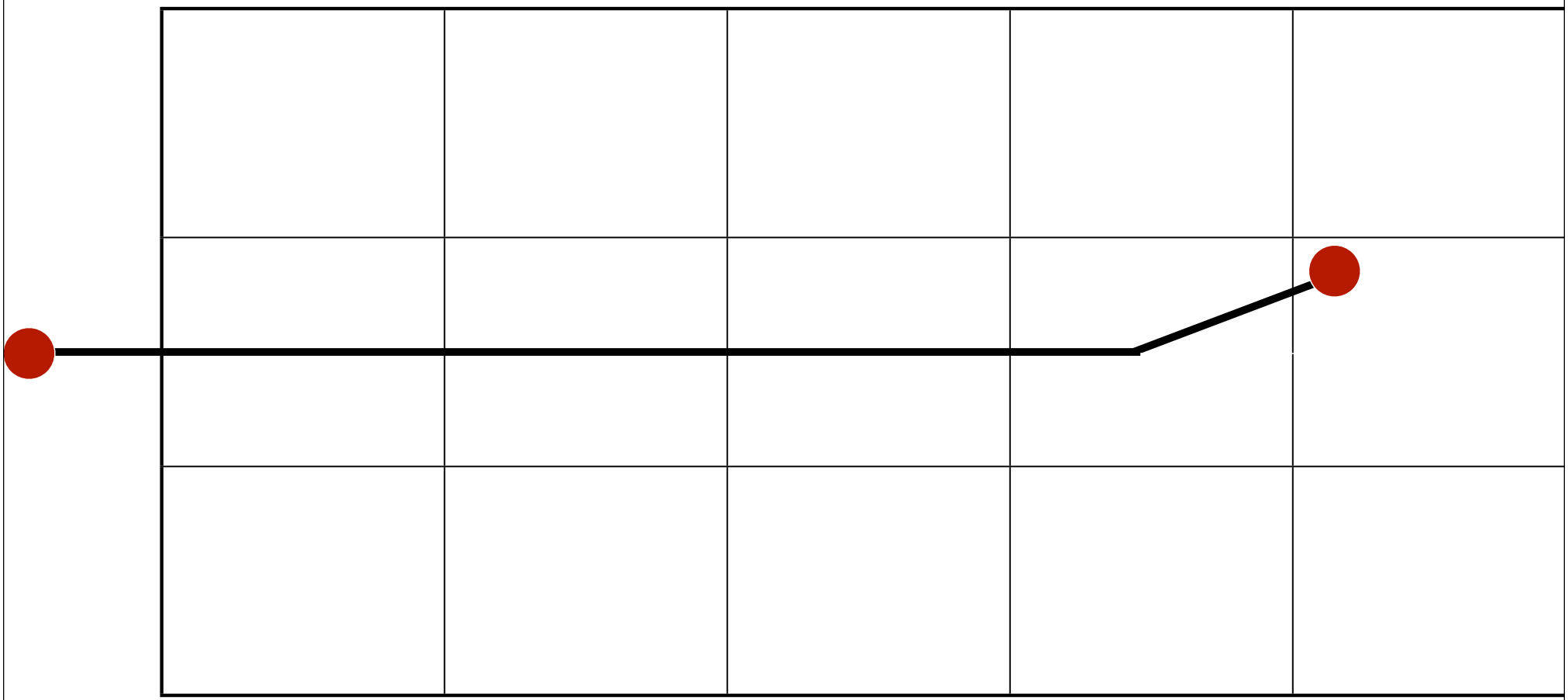
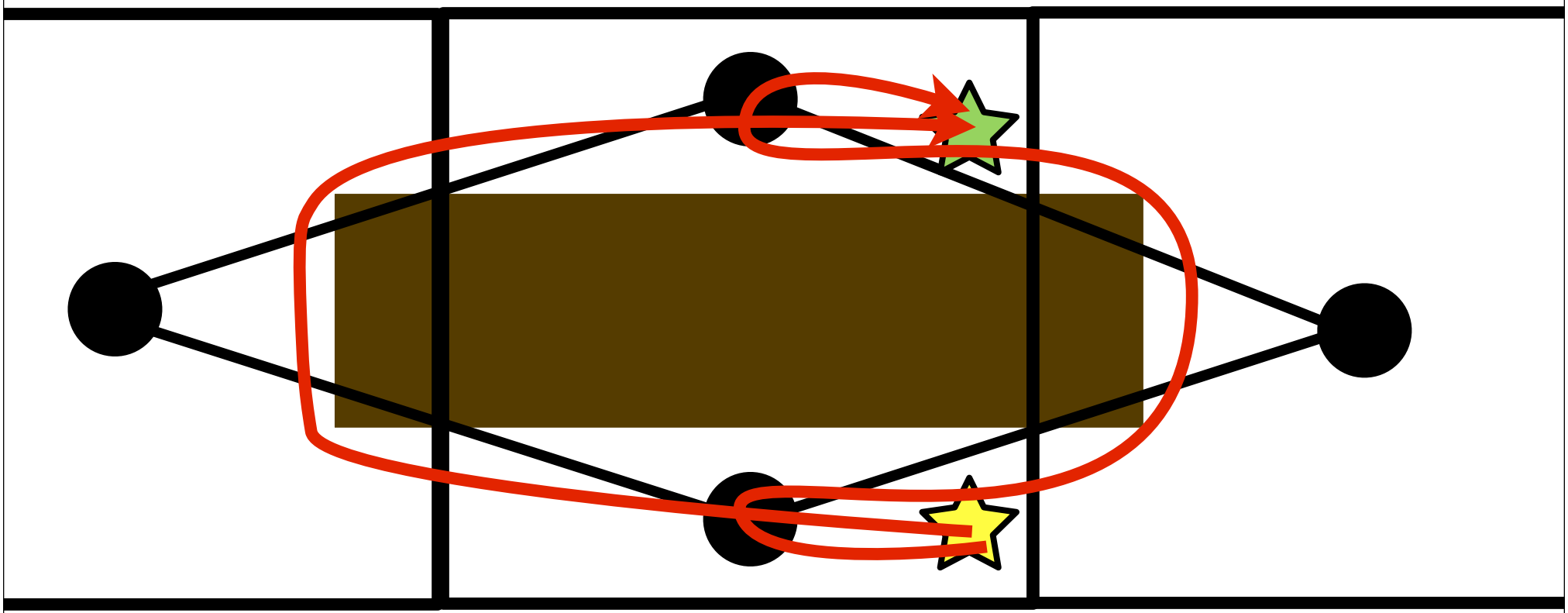
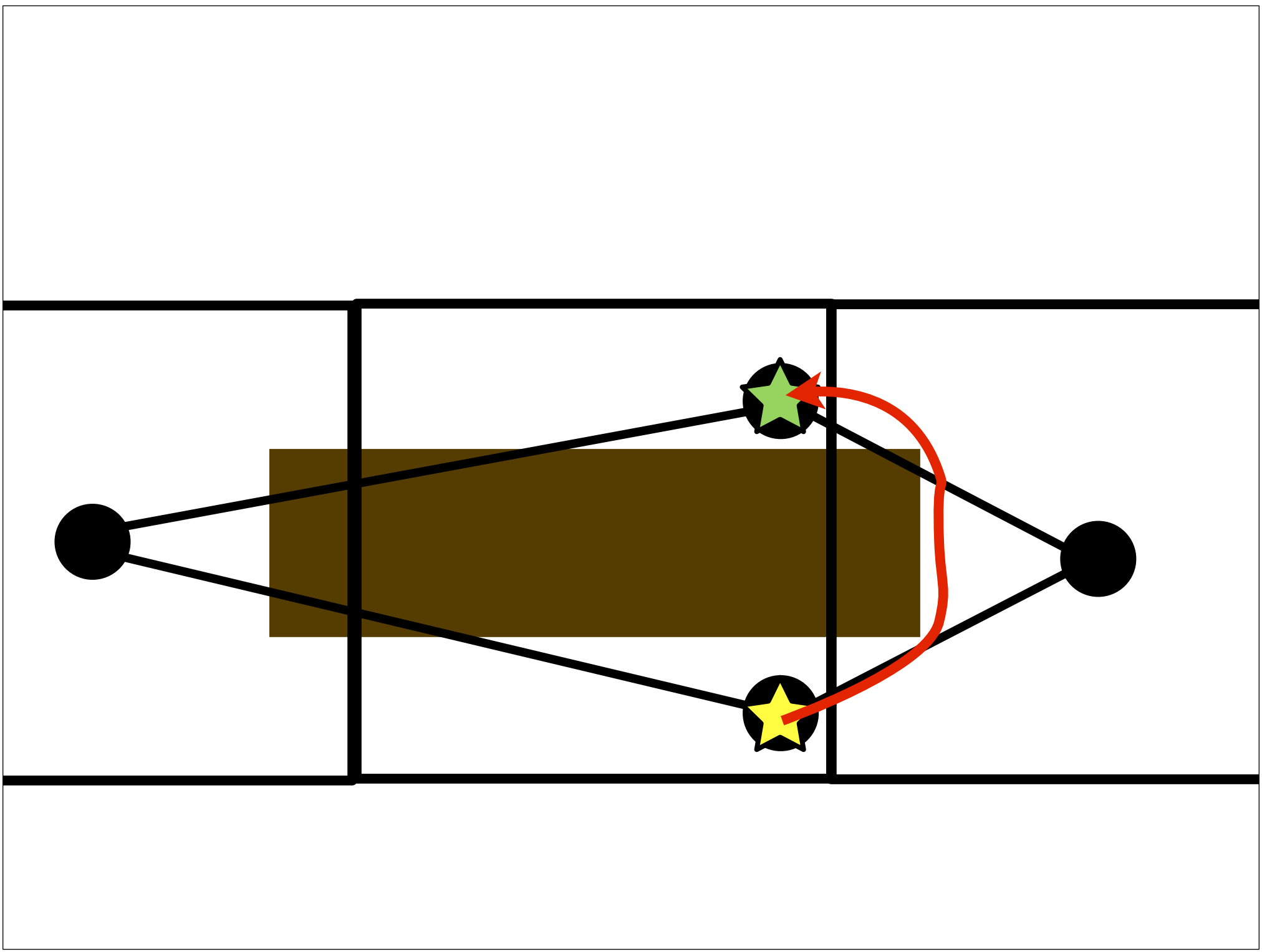
Orgrimmar Hall of

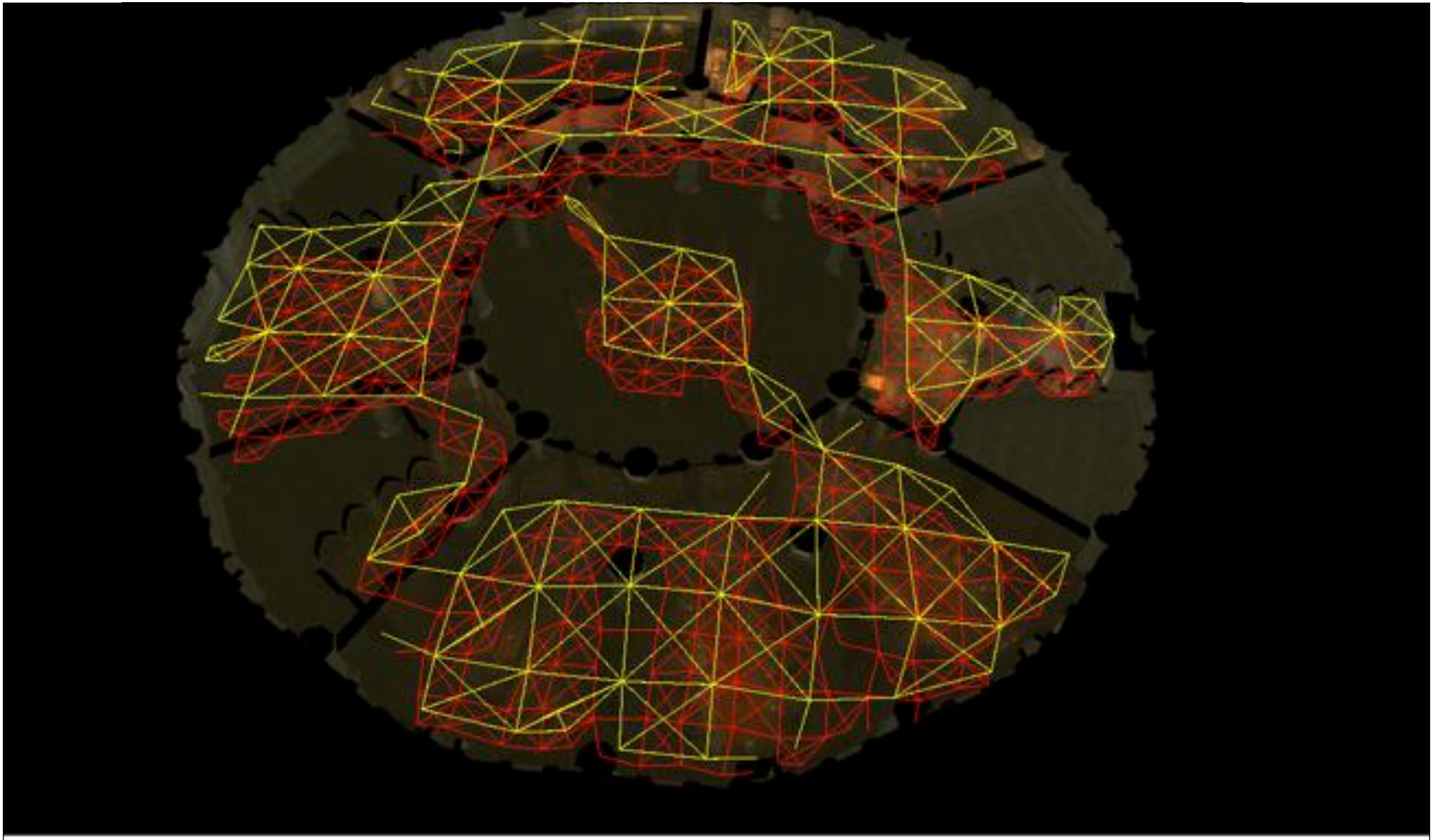




Solution -- end one step early and then go to goal

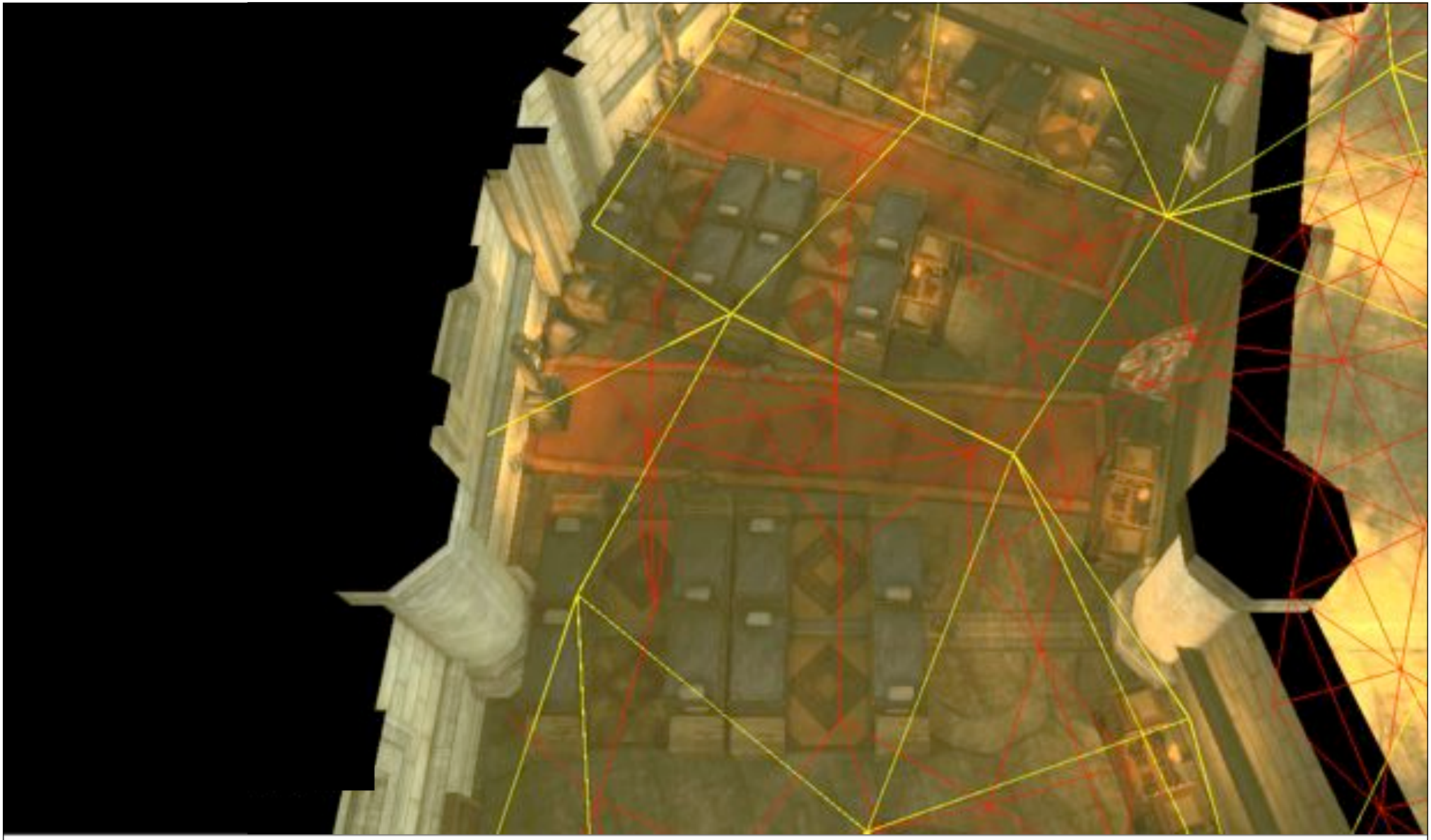






Dragon Age Pathfinding

Indoor Map



Dragon Age Pathfinding

Indoor Map: Detail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Apprentice Quarters


Orgrimmar Hall

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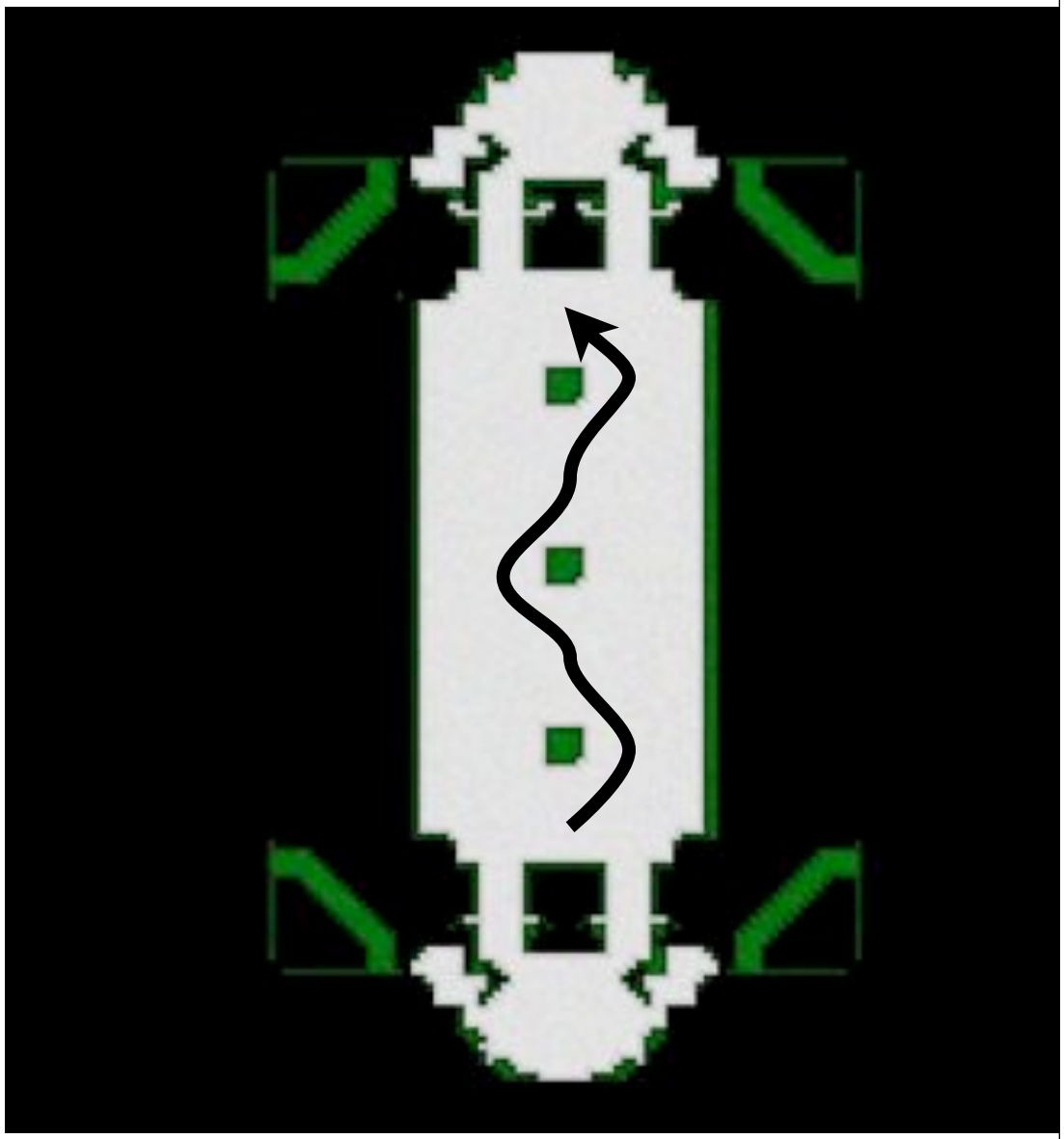
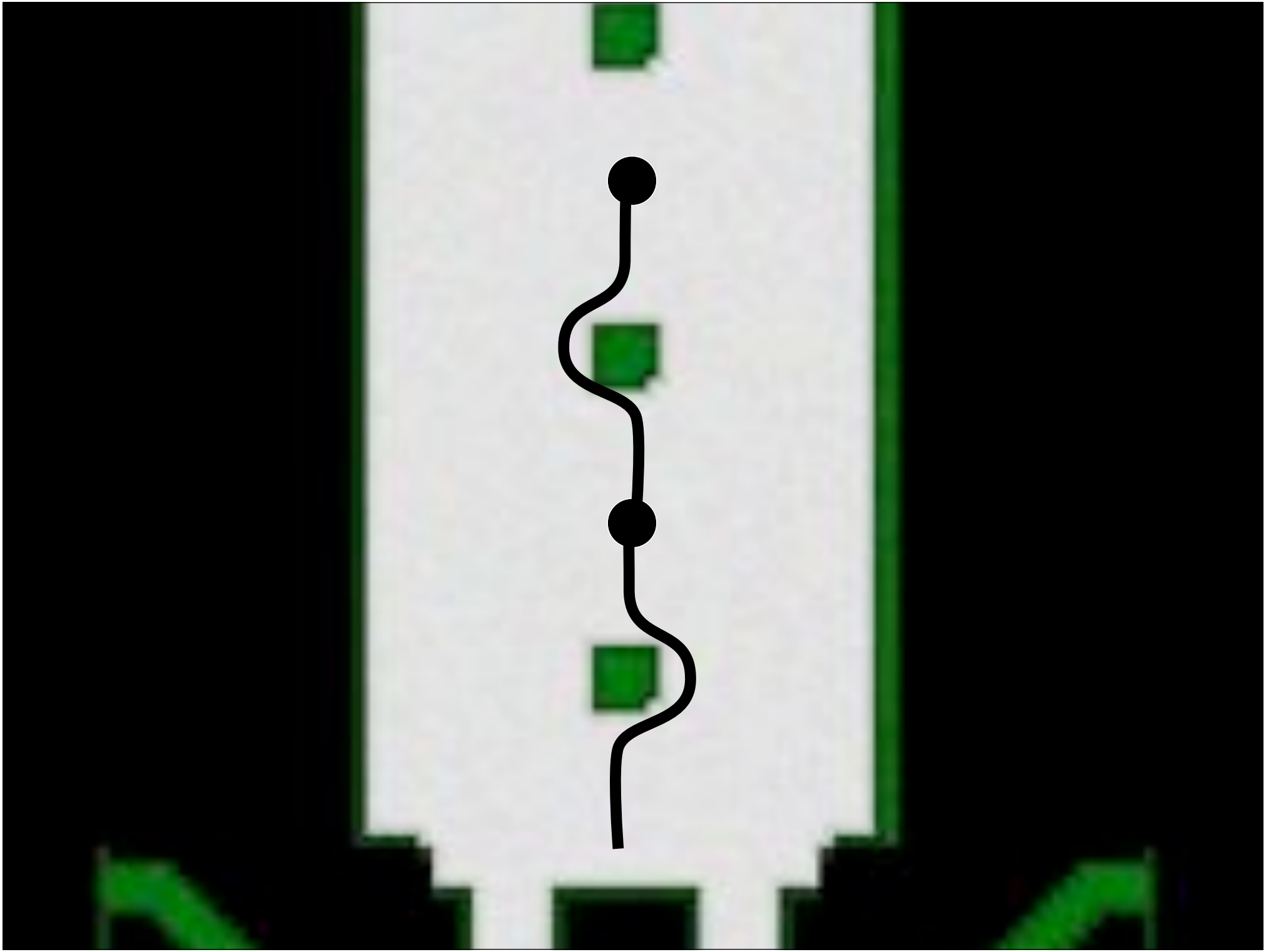
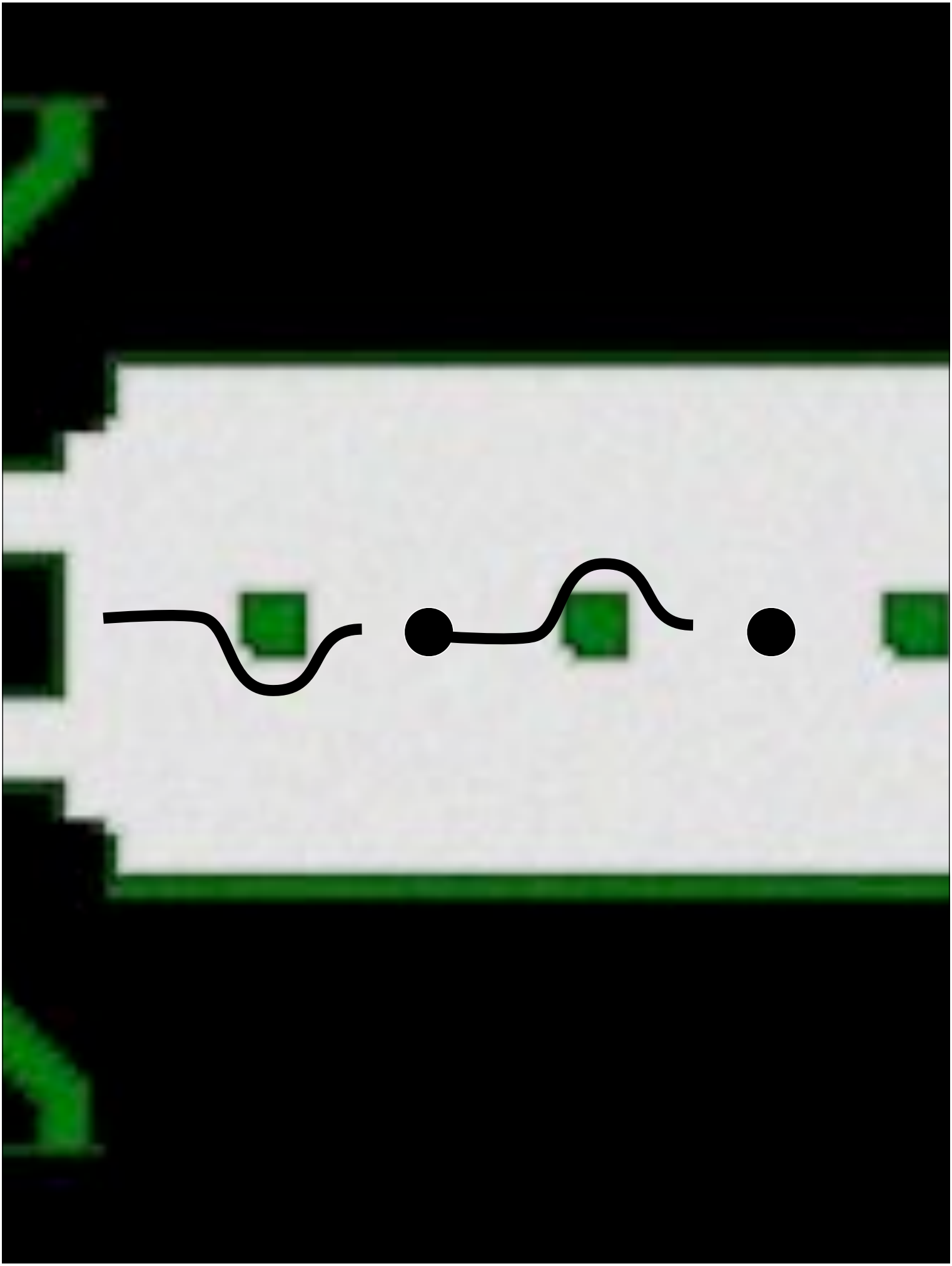
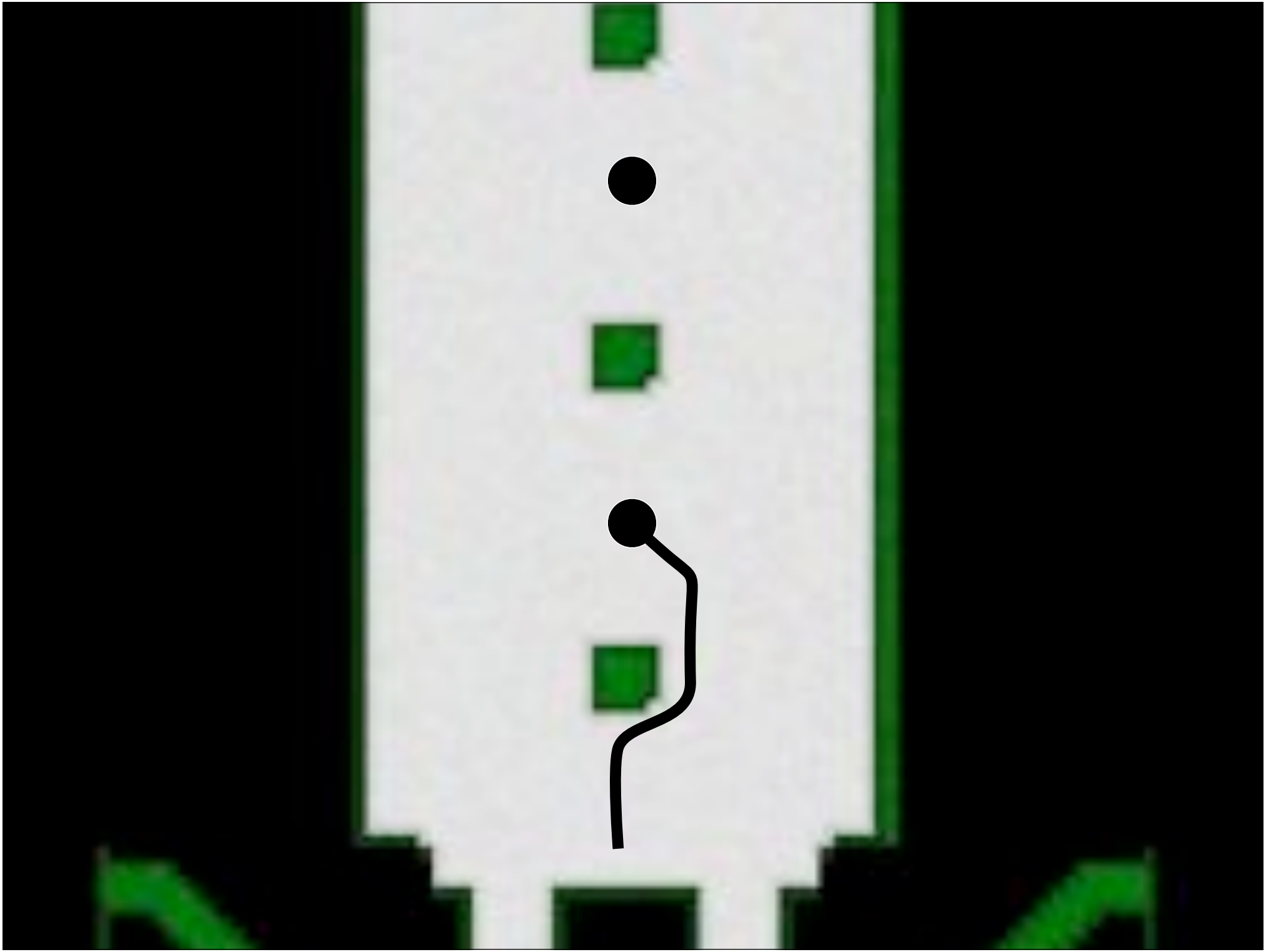
Path Continuity

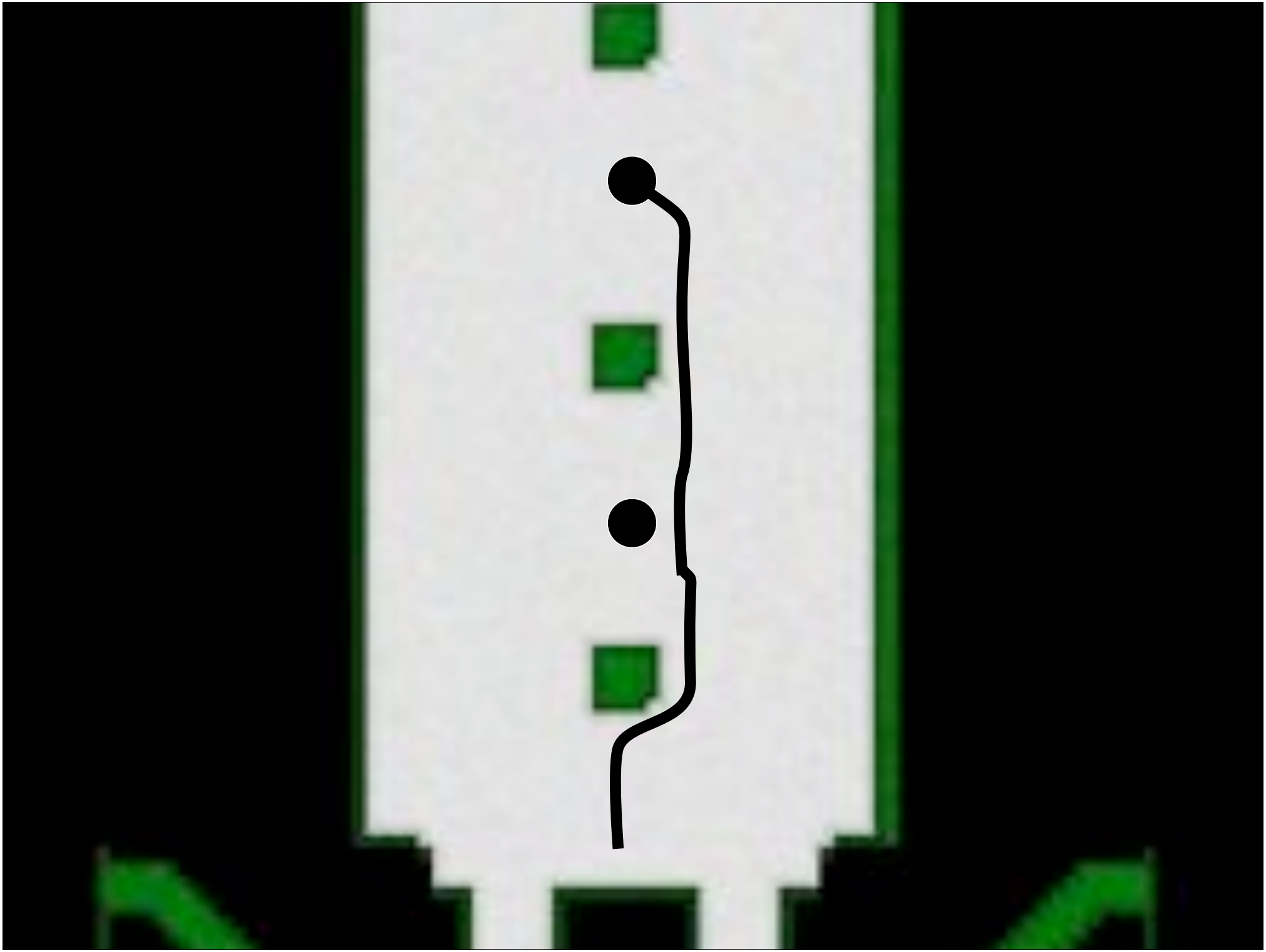
- Orzammar
- NPC doesn’t remember movement history

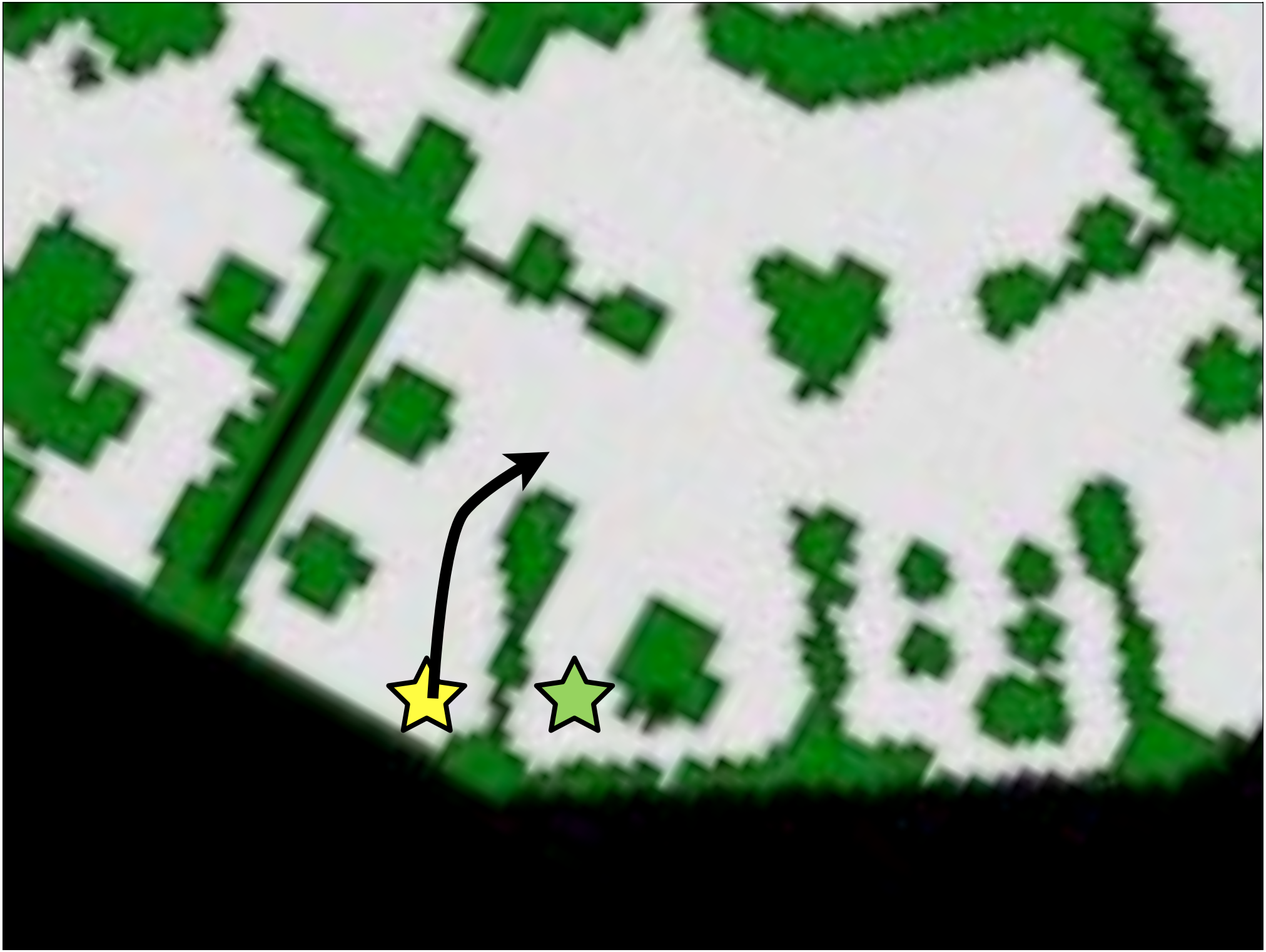


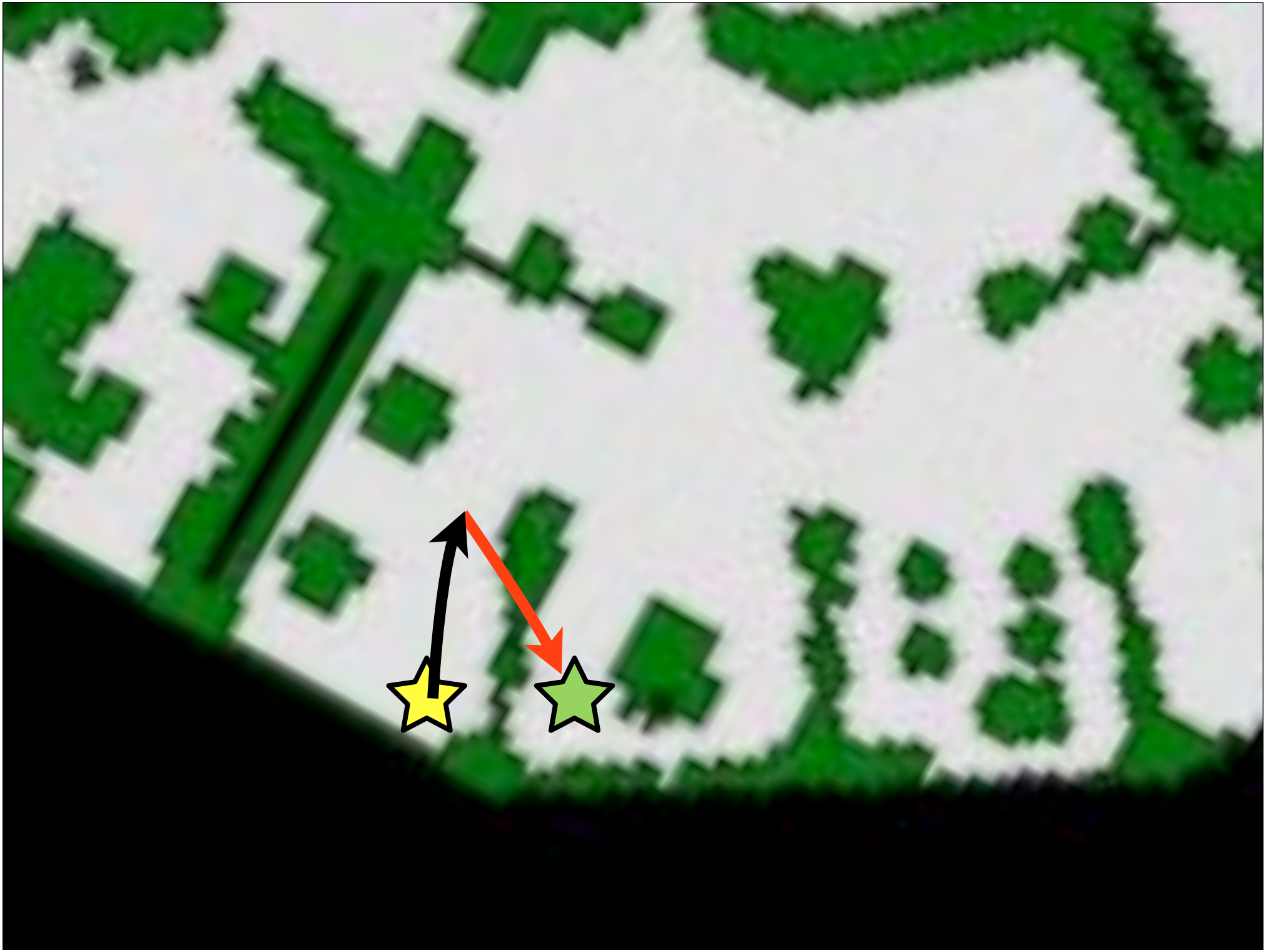












Heuristics

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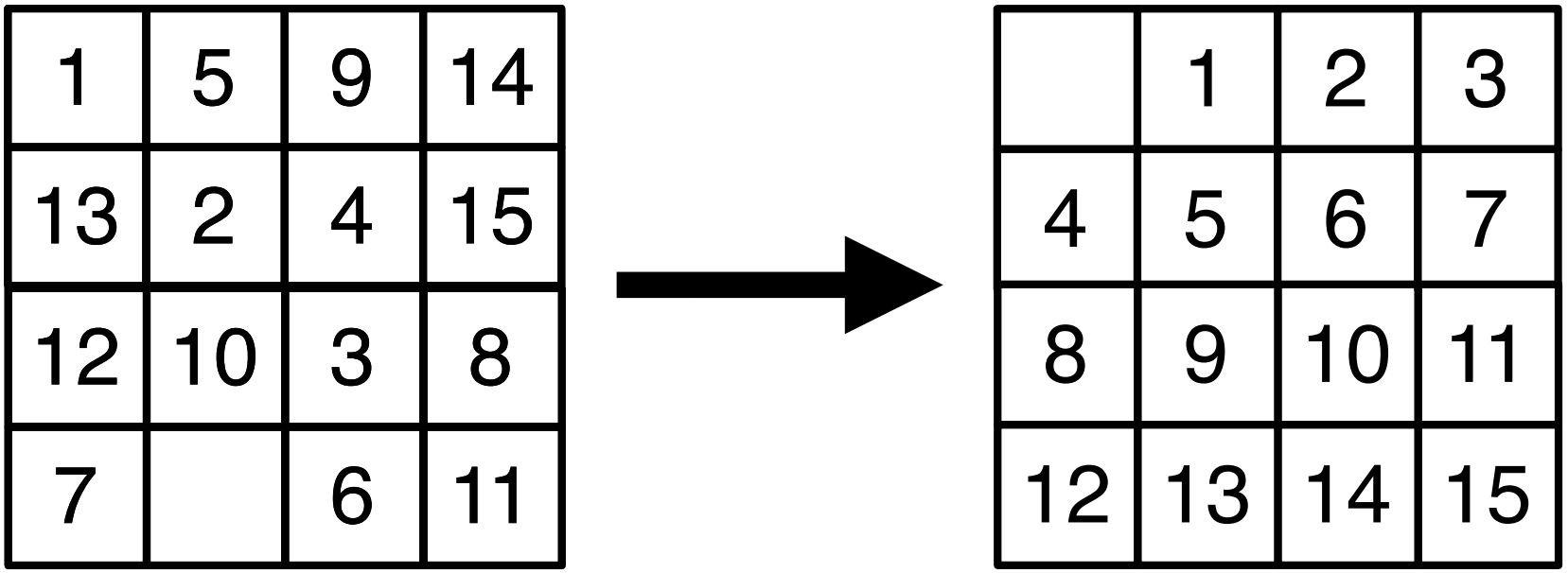
Heuristics

- Background
  - Pattern Databases
- True-Distance Heuristics

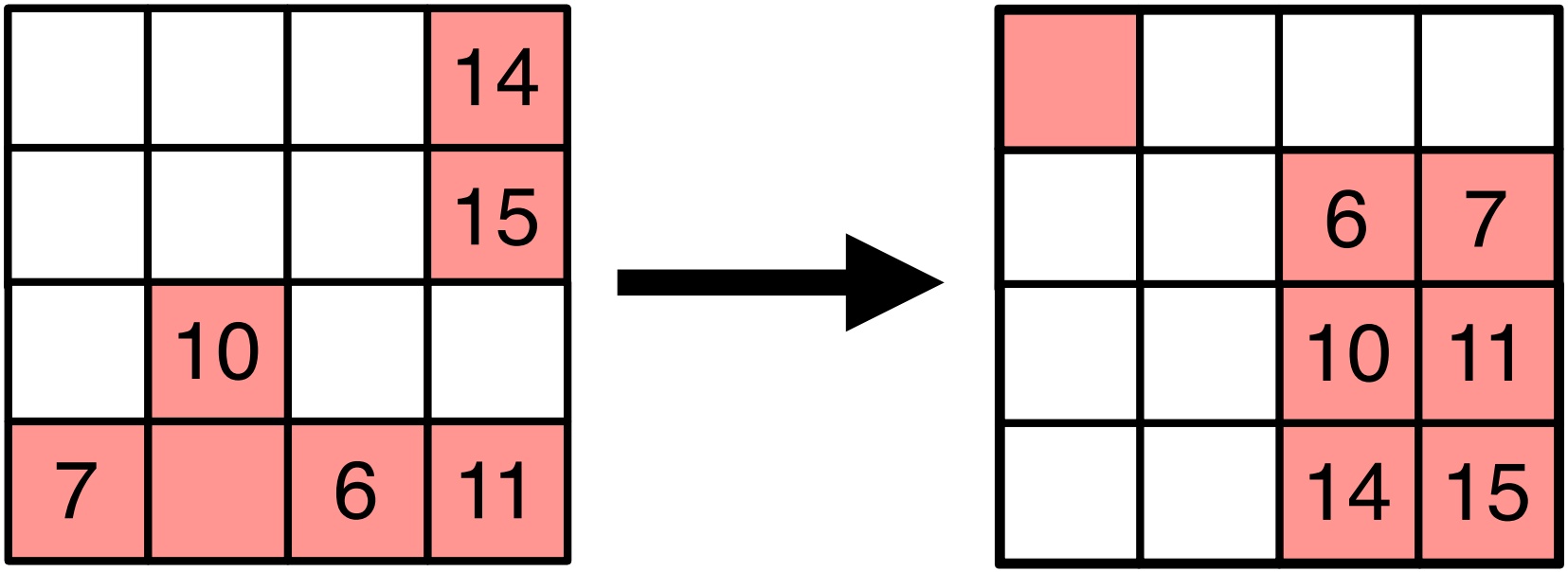
Previous work

- Pattern Databases (PDB)
  - Pre-computed, memory-based heuristic



Previous work - PDB

- Mark some states as “don’t care”
  - Creates abstract state space
  - Solve all states in abstract state space exactly



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PDB -- analysis

- PDBs have been applied to state spaces which grow exponentially
  - Branching factor b
  - Search depth d
- State space size $O(b^d)$

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PDB -- analysis

- Assume we can store 1/*f* of the state space
- Assume abstract branching factor is the same
- Let *c* be the max. dist. in the abstract state space
  - $b^c = 1/f \cdot b^d$
  - $c = d - \log_b(f)$
- Heuristics only lose a small amount of accuracy
  - 60MB 15-puzzle PDB has max value of 54
  - 10TB state space has max value of 80

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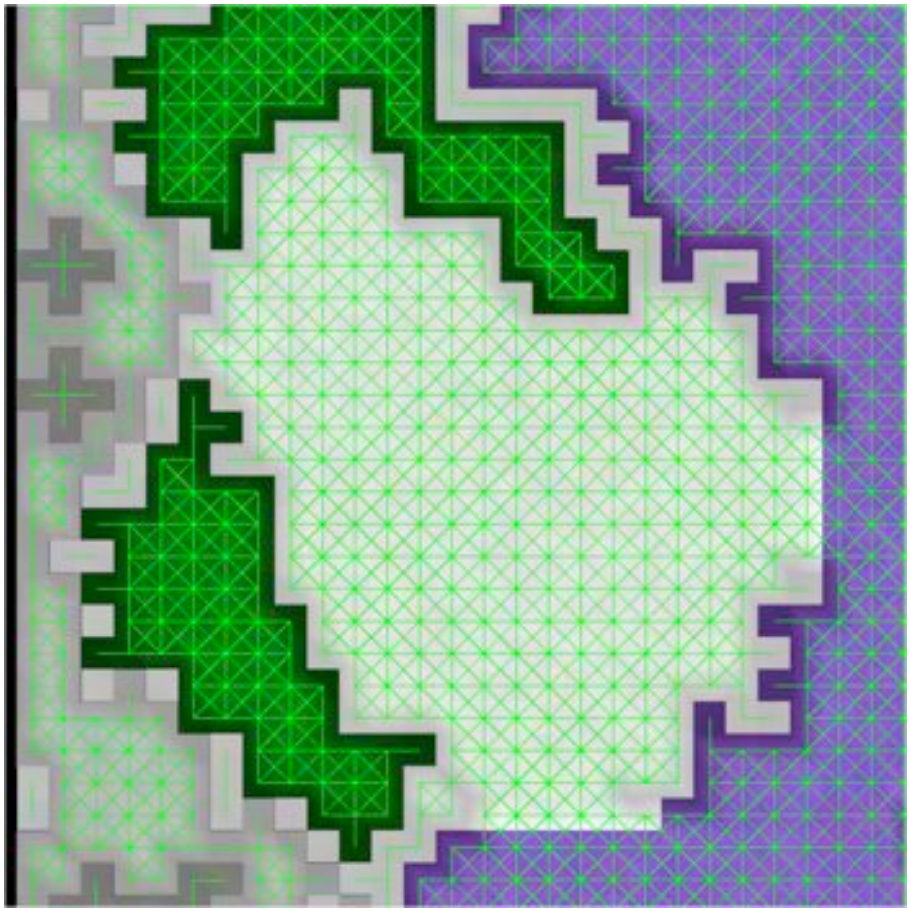
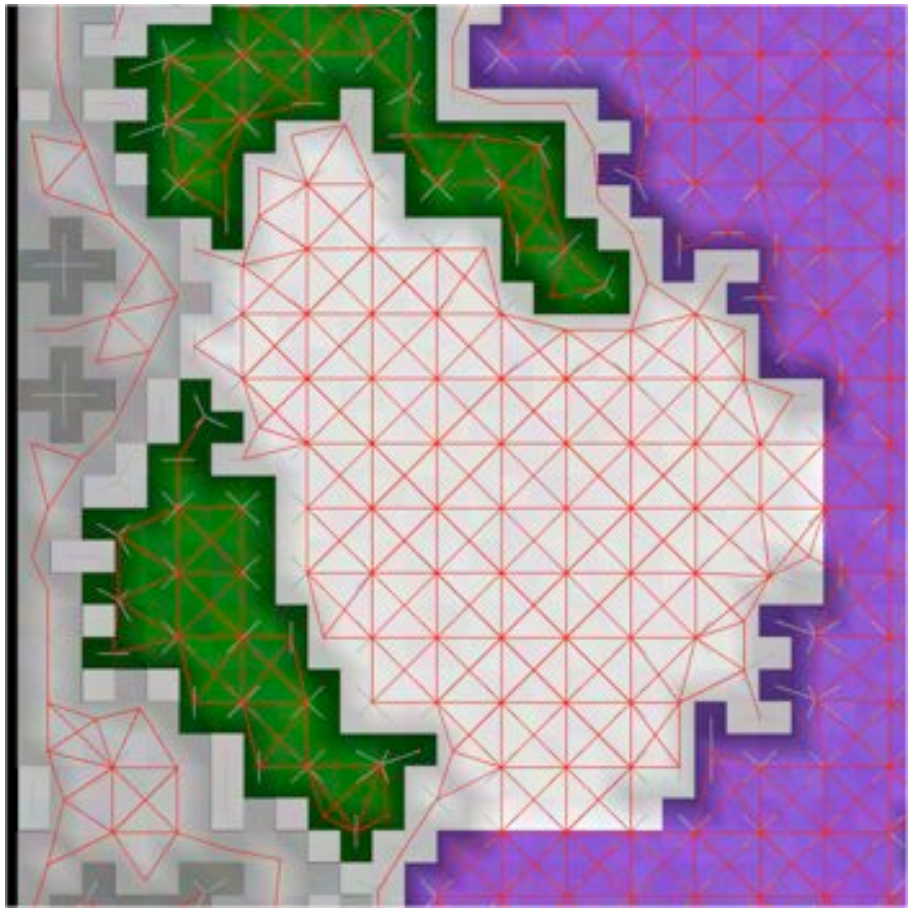
Abstraction in Maps



11,614 states



3,455 states

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Analysis -- maps

- 2D maps grow with r^2 (*r* is radius of search)
- If we can store $1/f$ of the full state space
  - $c^2 = 1/f \cdot r^2$
  - $c = r/\sqrt{f}$
- Abstract search space by factor of 4
- Maximum heuristic value is reduced by factor of 2
  - Bad for heuristics!

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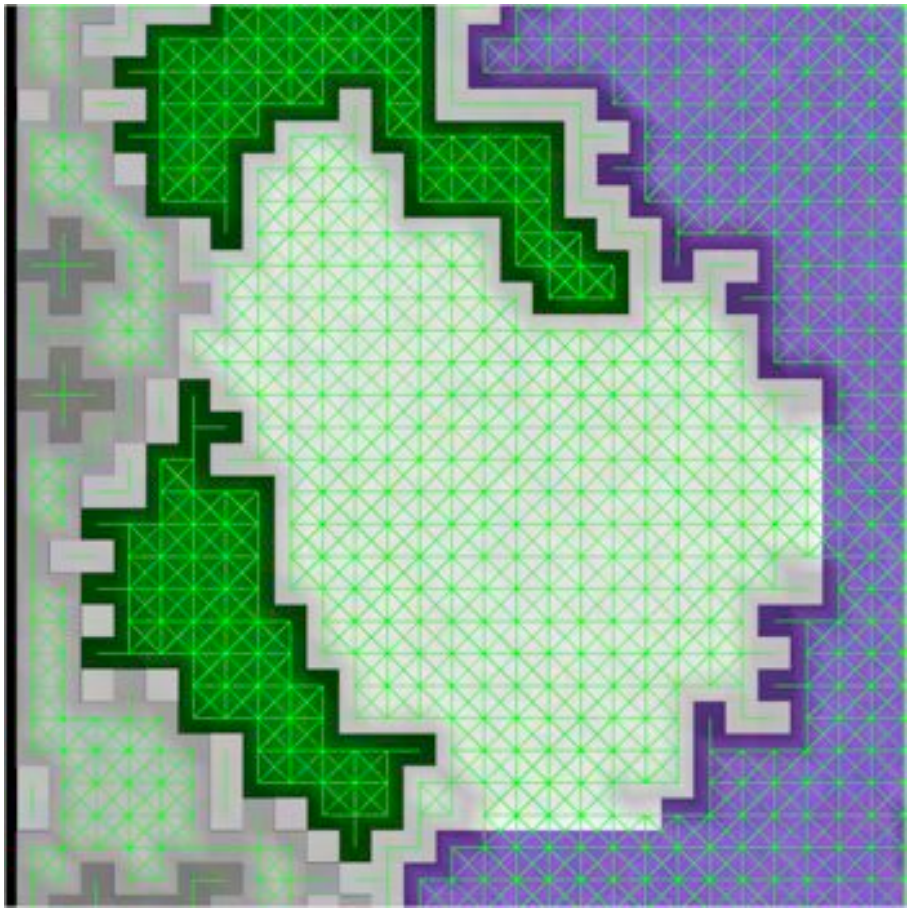
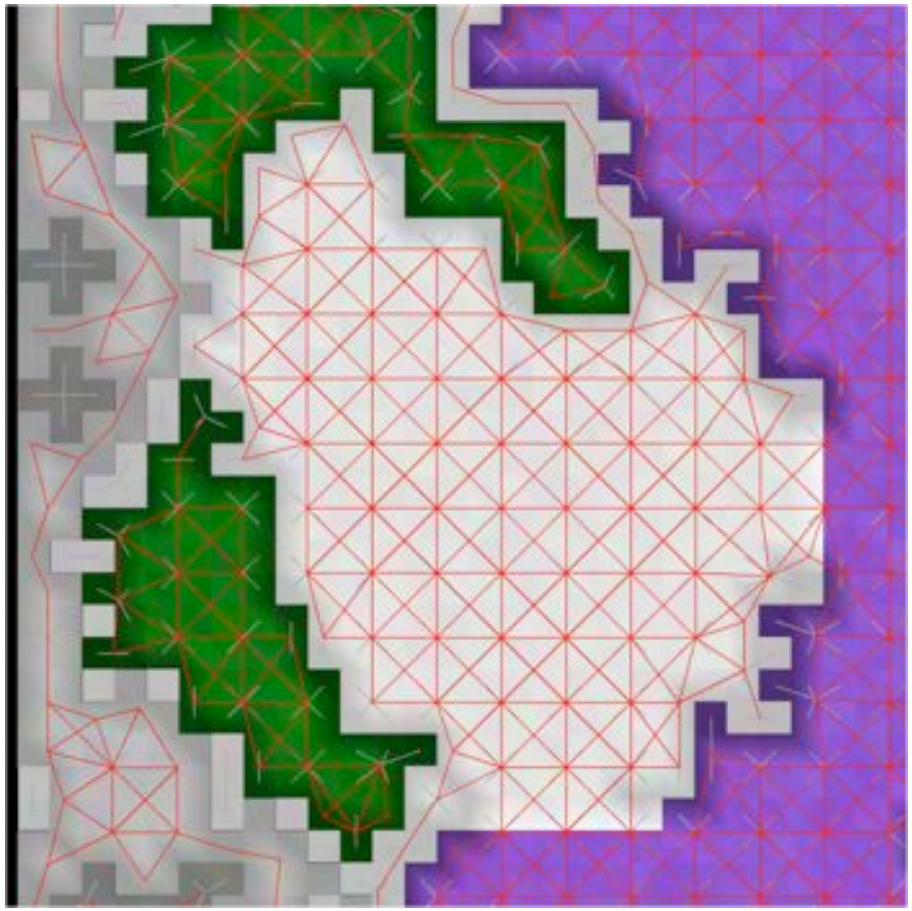
Abstraction in Maps



11,614 states
Width: 157



3,455 states
Width: 80

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Ideals

- Want a heuristic that can be used between any two states
- Want to minimize cost of the heuristic
- Heuristic should be based on true-distances in the world instead of abstract distances
  - New class, True-Distance Heuristic

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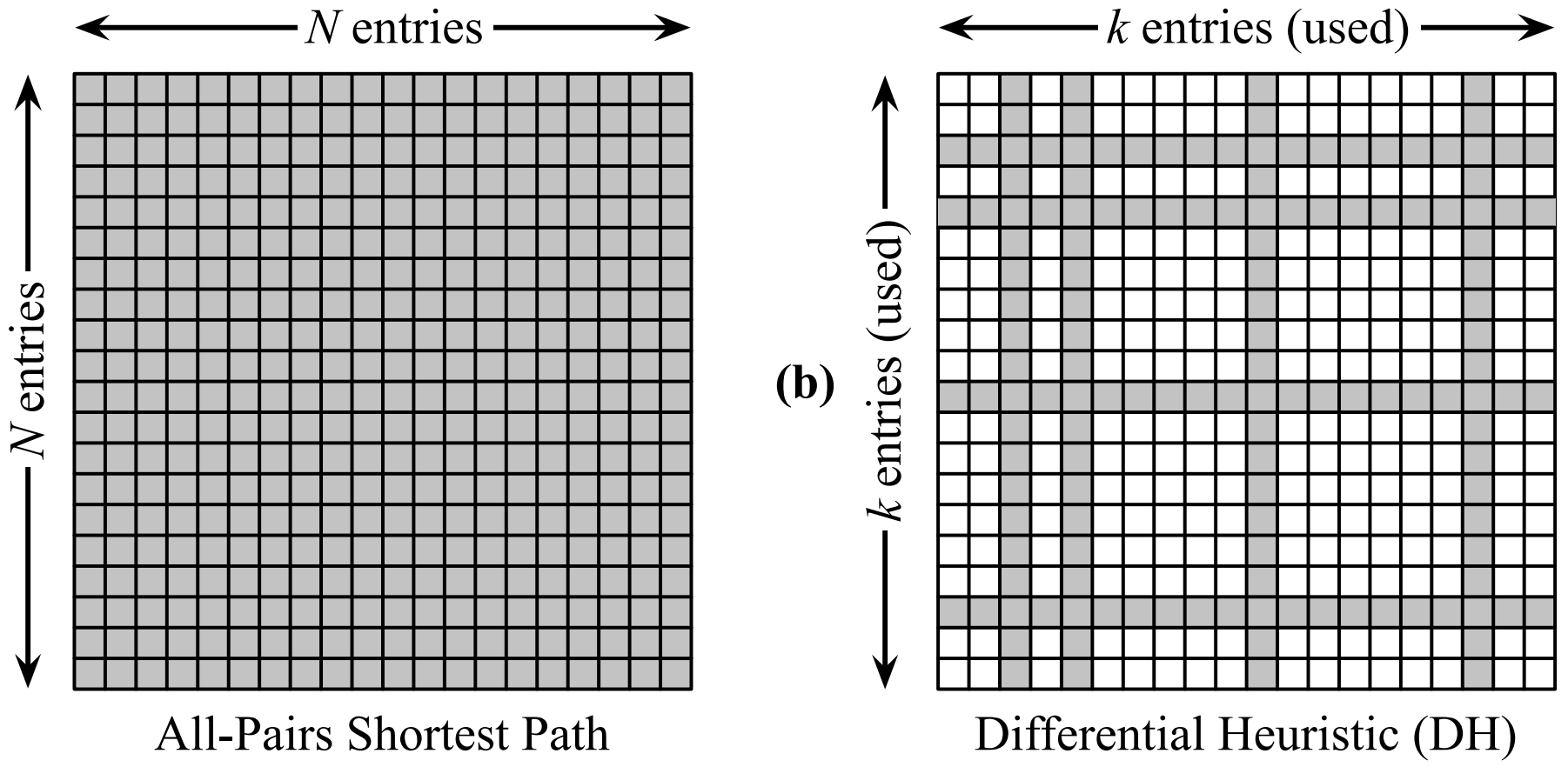
Differential Heuristic (ALT)

- If we have a solution to the single-source shortest-path problem for a state *s*, we can use it to get a heuristic between any two states
  - Assume undirected graph
  - h(a, b) = | d(a, *s*) - d(b, *s*) |
- Invented and re-invented in several different communities

Differential Heuristic

- Need to store multiple heuristics and take the max to get good results



A* - 10 diff. heuristics

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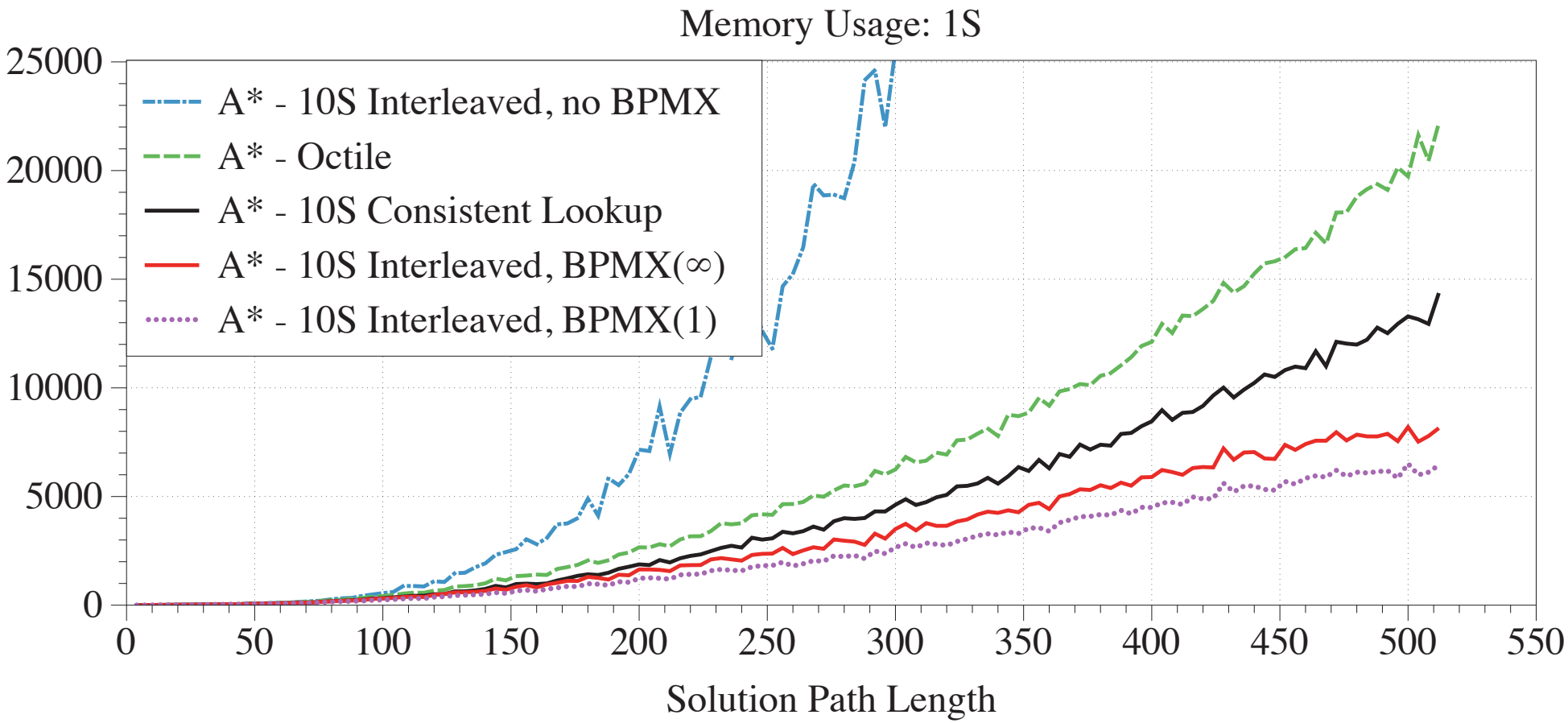
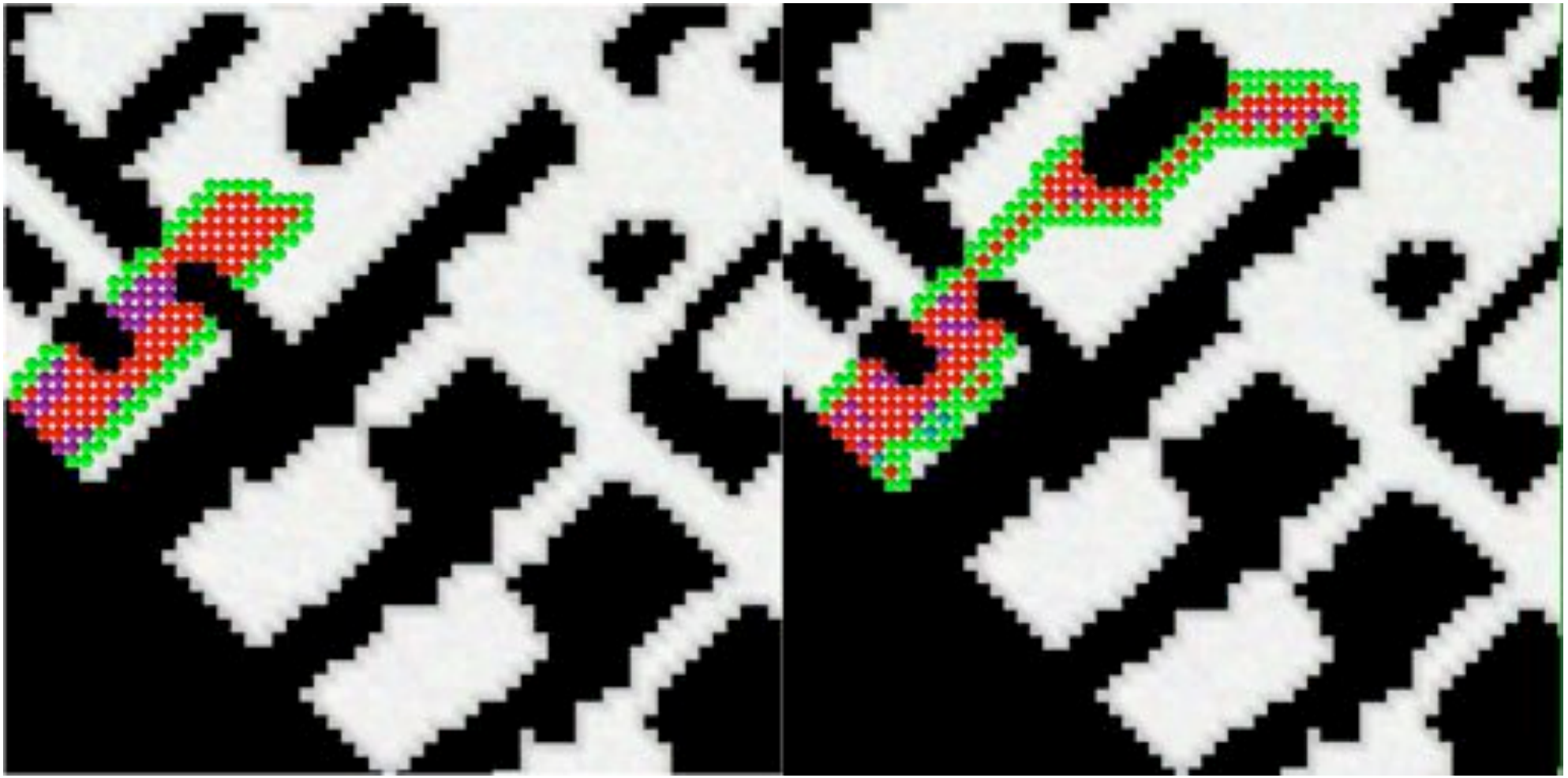
Improving performance

- What if we don't have much memory available?
- Can the heuristic be compressed?
  - Only store heuristics at some nodes
  - Goal is fixed, so find all heuristics around the goal
  - During search use whatever heuristic is available
    - Subtract distance from goal



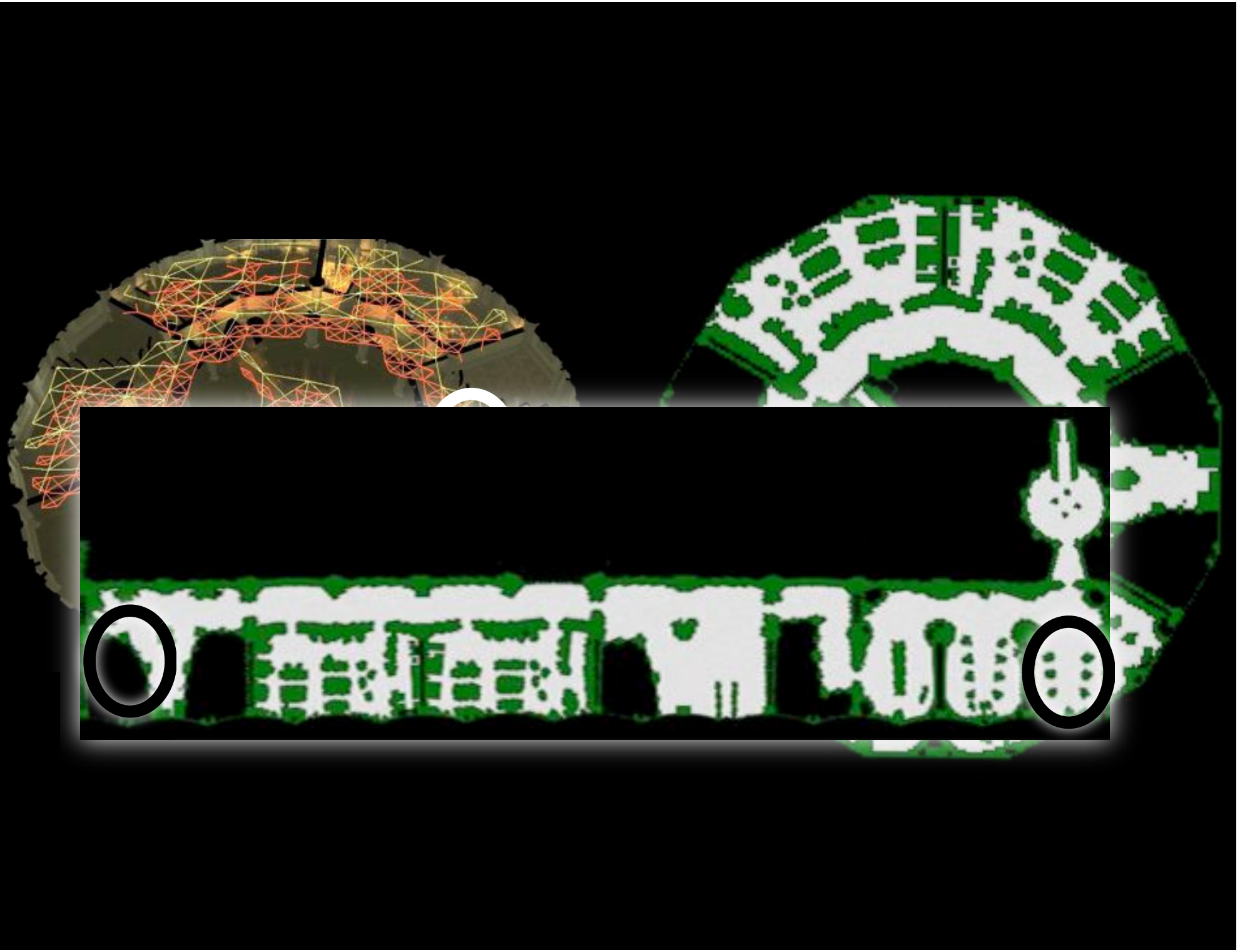
Inconsistent heuristics in theory and practice Ariel Felner, Uzi Zahavi, Robert Holte, Jonathan Schaeffer, Nathan Sturtevant, Zhifu Zhang, Artificial Intelligence

The Compressed Differential Heuristics
Meir Goldenberg, Nathan Sturtevant, Ariel Felner, Jonathan Schaeffer, AAAI 2011



Building Heuristics and Maximum Variance Unfolding

Euclidean Heuristic Optimization, Chris Rayner, Michael Bowling, Nathan Sturtevant, AAAI 2011



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Euclidean heuristics

- Euclidean heuristics are heuristic values that can be computed as distances in some Euclidean space of d dimensions
  - $h(i, j) = \|y_i - y_j\|$
- Let Y be the n by d matrix storing the vectors y_i. Y implicitly encodes the heuristic function.
- Can we find the best Euclidean heuristic?

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Finding Euclidean Heuristics

$$\underset{Y}{\textbf{minimize}} \quad \mathcal{L}(Y)$$

$$\textbf{subject to} \quad Y \text{ is admissible and consistent}$$

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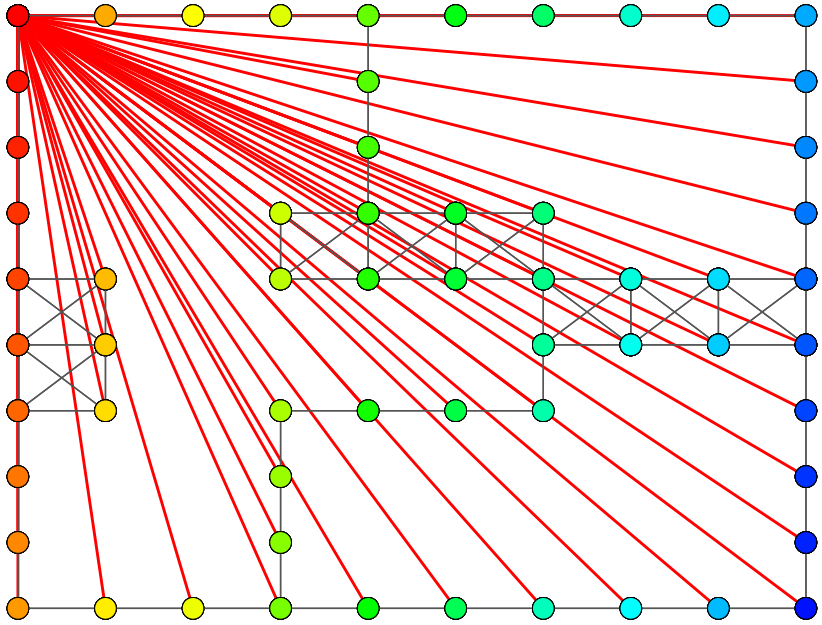
Expressing admissibility constraints

- Want to avoid pre-computing all paths and adding them as constraints in problem formulation



- Local consistency → global consistency → admissibility

Loss function

- Minimize squared error between true distance and heuristic
  - Weight coordinates *i*, *j* which are more important

$$\mathcal{L}(Y) = \sum_{i,j} W_{ij} \left| d(i,j)^2 - \|y_i - y_j\|^2 \right|$$

- Reformulate as:

$$\textbf{maximize}_{Y} \sum_{i,j} W_{ij} \|y_i - y_j\|^2$$

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Optimization problem

- Weighted generalization of MVU [Weinberger et al 2006] (for nonlinear dimensionality reduction)
  - Links learning a heuristic to manifold learning
  - Two separate areas which haven't been connected before
- A differential heuristic is a **one-dimensional embedding** in Euclidean space
  - Previously had heuristic methods for choosing where to place heuristics
  - Now have formal optimization

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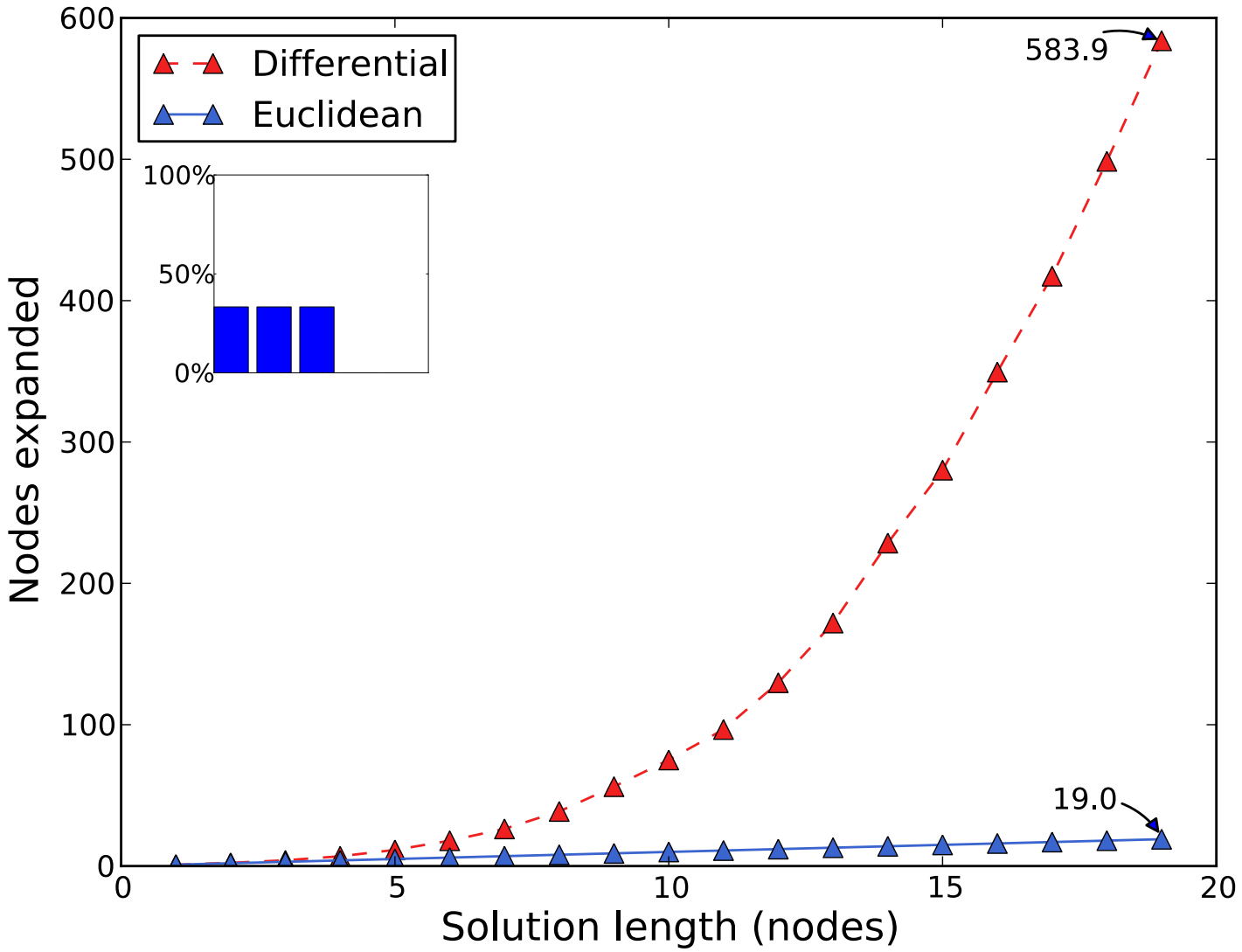
Example usage: 3-dimensional cube



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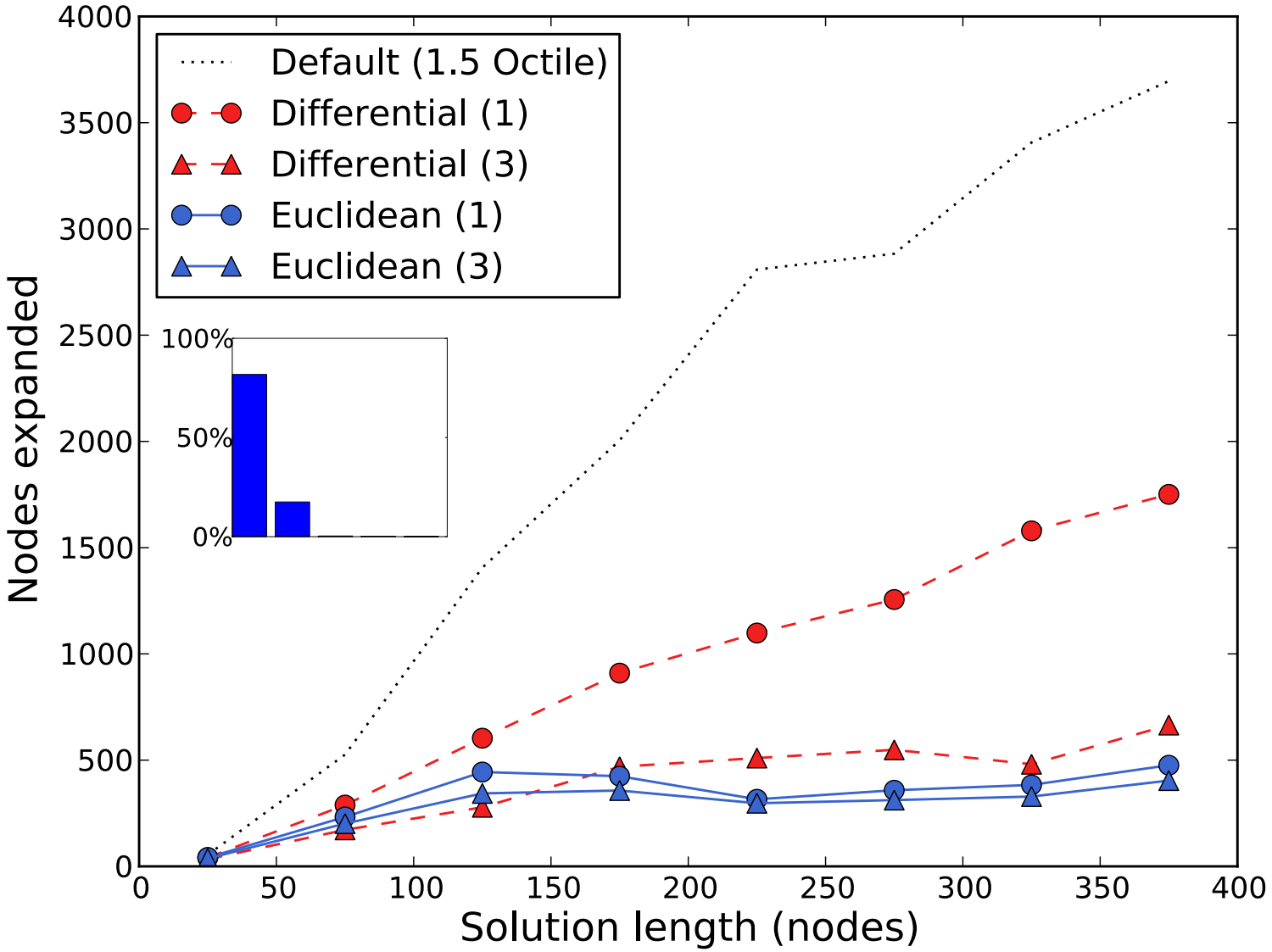
Example: Dragon Age Maps



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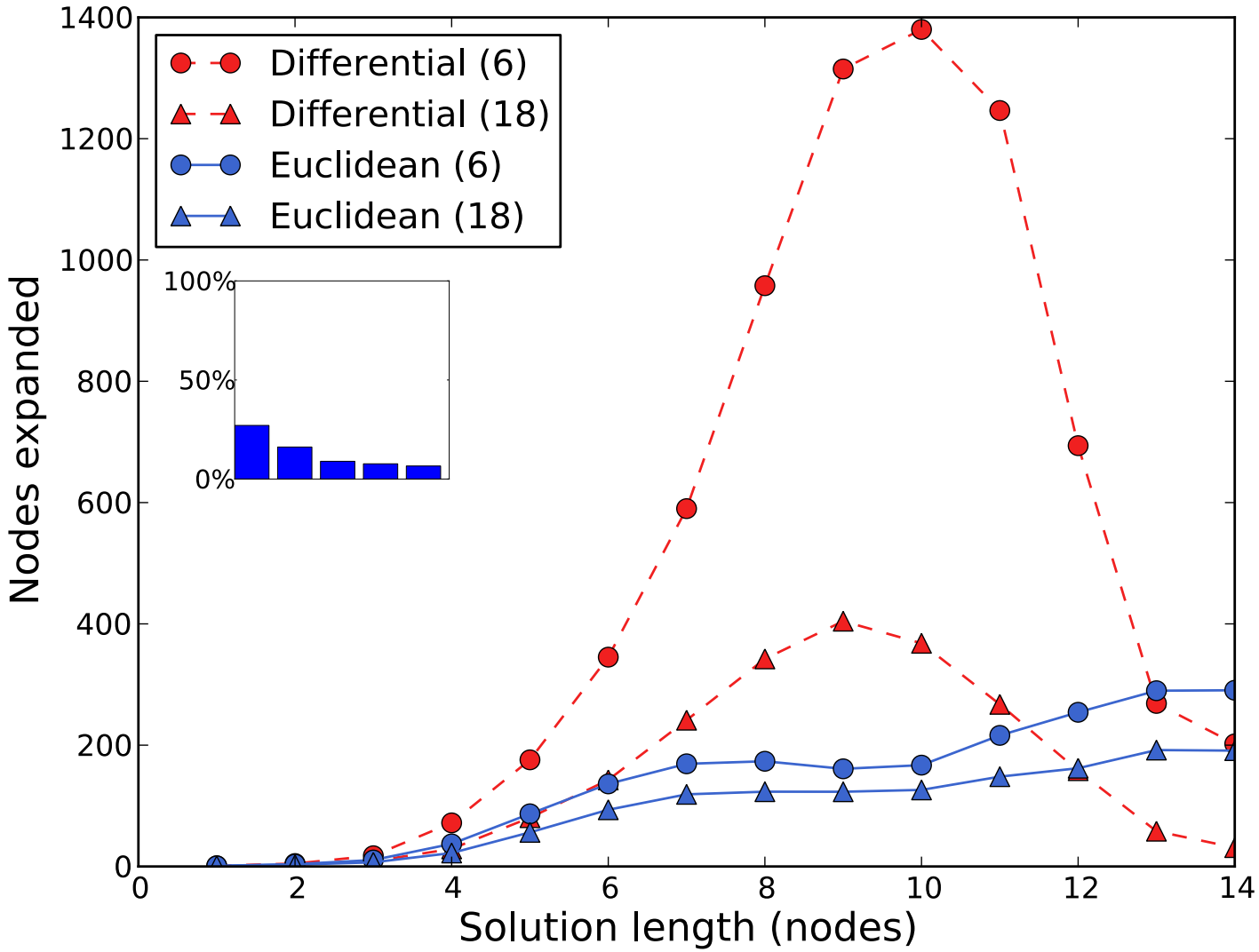
Example: Word search (edit distance)



FOUR
FOUL
FOIL
FAIL
FALL
FILL
FILE
FIVE

Summary

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Summary

- Looking for paths through state space
- Abstraction and heuristic techniques used to speed search
  - Availability of technique depends on underlying problem characteristics
- Large class of interesting problems which fit in memory

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Many other areas of search

- Planning
  - Building better heuristics from logic description of problem
- Randomizing search
  - Multiple OPEN lists
  - Monte-Carlo variations
- Suboptimal search